

The evolution of massive black holes and their spins in their galactic hosts

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ABSTRACT

Future space-based gravitational-wave detectors, such as LISA/SGO or a similar European mission (eLISA/NGO), will measure the masses and spins of massive black holes up to very high redshift, and in principle discriminate among different models for their evolution. Because the masses and spins change as a result of both accretion from the interstellar medium and the black-hole mergers that are expected to naturally occur in the hierarchical formation of galaxies, their evolution is inextricably entangled with that of their galactic hosts. On the one hand, the amount of gas present in galactic nuclei regulates the changes in the black-hole masses and spins through accretion, and affects the mutual orientation of the spins before mergers by exerting gravito-magnetic torques on them. On the other hand, massive black holes play a central role in galaxy formation because of the feedback exerted by AGN activity on the growth of structures. In this paper, we study the mass and spin evolution of massive black holes within a semianalytical galaxy-formation model that follows the evolution of dark-matter halos along merger trees, as well as that of the baryonic components (hot gas, stellar and gaseous bulges, and stellar and gaseous galactic disks). This allows us to study the mass and spin evolution in a self-consistent way, by taking into account the effect of the gas present in galactic nuclei both during the accretion phases and during mergers. Also, we present predictions, as a function of redshift, for the fraction of gas-rich black-hole mergers – in which the spins prior to the merger are aligned due to the gravito-magnetic torques exerted by the circumbinary disk – as opposed to gas-poor mergers, in which the orientation of the spins before the merger is roughly isotropic. These predictions may be tested by LISA or similar spaced-based gravitational-wave detectors such as eLISA/NGO or SGO.

Key words: supermassive black holes – spin – numerical relativity – gravitational waves – LISA – eLISA – NGO – galaxy formation

1 INTRODUCTION

Massive black-hole (MBH) mergers are expected to be the brightest sources of gravitational waves for future space-based detectors such as the Laser Interferometer Space Antenna (LISA) (Bender et al. 1998; Danzmann Rüdiger 2003; The LISA International Science Team 2011) or a similar mission led by ESA (eLISA/NGO, see Amaro-Seoane et al. (2012); Jennrich et al. (2012)) or NASA (SGO, see The SGO Core Concept Team (2011)). These detectors are expected to be capable of observing tens or even hundreds of merger events during their lifetime, up to redshift $z \sim 10$ or larger (Sesana, Volonteri, & Haardt 2007; Sesana et al. 2011; Arun et al. 2009), and measure black-hole masses and spins with astonishing accuracy ($\sim 10^{-3}$ for the masses and 10^{-2} for the spins, see Berti, Buonanno, & Will (2005);

Lang & Hughes (2006, 2007, 2008); Lang, Hughes, & Cornish (2011)). Also, they should be able to tell a binary of black holes with aligned spins from one with misaligned (and therefore precessing) spins by looking at the higher-order harmonics of the gravitational waveforms (Lang & Hughes 2006, 2007, 2008; Lang, Hughes, & Cornish 2011).

Massive black holes are also expected to play a crucial role in galaxy formation: in fact, in Active Galactic Nuclei (AGNs) accretion onto the central MBH is believed to power jets or disk winds capable of exerting a feedback on the growth of structures, by ejecting gas from the interstellar medium (ISM) and from the intergalactic medium (IGM) (Granato et al. 2004; Lapi et al. 2006; Croton et al. 2006; Bower et al. 2006; Hopkins et al. 2008), therefore quenching star formation in low-redshift, massive galaxies. Indeed, this feedback mechanism is a central ingredient of our current understanding of galaxy formation, because it helps explaining why large dark-matter halos present low baryonic masses, resulting in a sharp cutoff at the high-mass end of the stellar mass

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function that is not observed in the halo mass function (Bell et al. 2003a; Benson et al. 2003). Also, it helps making sense of the so-called “anti-hierarchical” evolution (or “downsizing”) of baryonic structures, i.e. the fact that the most massive galaxies are dominated by old stellar populations, while low-mass galaxies generally present young stellar populations and longer-lasting star-formation activity (Cowie et al. 1996; Fontanot et al. 2009), thus suggesting that massive galaxies assemble at higher redshift than low-mass ones. Naively, this behavior may seem in contrast with the “bottom-up” formation of dark-matter halos, which assemble hierarchically by a series of mergers, but it can be reproduced, at least in its main features, by semianalytical galaxy-formation models including the effect of AGN feedback (Scannapieco, Silk, & Bouwens 2005).

This link between MBHs and the larger-scale galactic properties works also in the opposite direction, because it is the amount of cold gas present in galactic nuclei that regulates accretion onto the MBHs, and therefore their mass and spin evolution. Also, the 100-pc scale circumbinary disks that are thought to form after gas-rich mergers of galaxies (Mayer et al. 2007) exert torques on the spins of the MBHs, aligning them by the time the binary’s separation has shrunk to ~ 0.1 pc (Bardeen & Petterson 1975; Bogdanović, Reynolds, & Miller 2007; Perego et al. 2009; Dotti et al. 2010a; Maio et al. 2012), and further spin alignment occurs from this separation to the merger as a result of Post-Newtonian resonances (Schnittman 2004; Kesden, Sperhake, & Berti 2010). Not only do these effects profoundly influence the spin and mass evolution – because if the black-hole spins are aligned before the merger, the spin of the final black hole is larger (Tichy & Marronetti 2008; Lousto et al. 2010; Rezzolla et al. 2008; Barausse & Rezzolla 2009; Buonanno, Kidder, & Lehner 2008; Kesden 2008) and its mass lower (Tichy & Marronetti 2008; Reisswig et al. 2009; Lousto et al. 2010; Kesden 2008) – but they also affect whether the MBH resulting from the merger remains in the galaxy or gets ejected. In fact, numerical-relativity simulations of black-hole binaries have shown that mergers can produce final black holes with large kick velocities relative to the center of mass of the initial binary configuration, due to anisotropic emission of gravitational waves. In particular, for equal-mass configurations with initial black-hole spins lying on the orbital plane of the binary, the kick velocity can be as large as 2500–4000 km/s (Campanelli et al. 2007; González et al. 2007), and velocities as high as 5000 km/s may occur for configurations with off-equatorial spins (Lousto & Zlochower 2011; Lousto et al. 2012). Such velocities are larger than the typical galaxy escape velocities, thus leading to the staggering conclusion that most galaxies may not host a MBH. This would be in stark contrast with observations that most galaxies do host a MBH at low redshifts, and it may be bad news for hierarchical galaxy formation models, which as mentioned above heavily rely on the feedback from MBHs. However, for binaries with aligned spins the kick velocity is considerably lower and typically not sufficient for ejecting the final black hole from the galaxy. Because, as mentioned above, circumbinary disks tend to align the black-hole spins prior to mergers, it is clear that tracking the evolution of gas in galaxies is crucial to correctly predict how many galaxies host a MBH, let alone the MBH mass and spin evolution.

What makes galaxy formation a difficult problem is the huge range of scales that are involved, which go from the Gpc scale of the present cosmological horizon and the Mpc scale of typical $z \sim 0$ galactic halos, through the 10-kpc scale of galactic disks and kpc scale of galactic spheroids, down to the 100-pc scale of

circumbinary disks and pc scale of MBH accretion disks, and finally to the 10^{-6} – 10^{-7} pc scale at which MBH mergers take place. This huge dynamical range, together with the complex nature of the processes, often nonlinear and dissipative, that take place on small scales (“subgrid physics”), makes the problem very difficult to solve numerically in full generality. In fact, while the current paradigm of cosmological structure formation (the Λ CDM model) has enjoyed remarkable success in reproducing large-scale observations [such as the cosmic microwave background fluctuations (Jarosik et al. 2011; Larson et al. 2011); the large scale clustering of galaxies (Komatsu et al. (2011); Eisenstein et al. (2005), and references therein); the cosmic shear field measured through weak gravitational lensing (Fu et al. (2008) and references therein); the small scale power spectrum of Lyman-alpha forest sources (Jena et al. 2005); the number density of galaxy clusters (Henry et al. (2009) and references therein)], a global understanding of galaxy formation can presently be attempted only by means of semi-analytical models (Kauffmann, White, & Guiderdoni 1993; Cole et al. 1994, 2000; Somerville & Primack 1999; Somerville et al. 2008; Croton et al. 2006; Bower et al. 2006; Benson & Bower 2010; Monaco, Fontanot, & Taffoni 2007), or more ambitiously with hydrodynamical simulations (Springel & Hernquist 2003; Springel 2005; Di Matteo, Springel, & Hernquist 2005; Scannapieco et al. 2009; Dotti et al. 2007; Dotti, Colpi, & Haardt 2006; Dotti et al. 2009, 2010a; Maio et al. 2012; O’Shea et al. 2004; Kravtsov & Gnedin 2005; Tassis et al. 2003; Tassis, Kravtsov, & Gnedin 2008; Dubois & Teyssier 2008; Blecha et al. 2011; Guedes et al. 2011; Sijacki, Springel, & Haehnelt 2011, 2009; Teyssier et al. 2011).

In this paper we use a semianalytical model for the formation of galaxies in a Λ CDM universe to study the evolution of the spins and masses of MBHs in a self-consistent way, taking into account both the feedback of the MBHs on the growth of structures and the influence of the galactic nuclear gas on the MBH accretion history and on the spin-alignment prior to mergers (which in turn affects the spin evolution, as well as the kick velocities of MBHs and thus their possible ejection from galaxies). For the purpose of this investigation, we adopt the widely accepted scenario in which the IGM collapses into disk structures, which give rise to bulges (“spheroids”) when disrupted by major (i.e. comparable-mass) galactic mergers, or when they become self-gravitating and develop bar-instabilities. To describe the evolution of dark-matter halos, we use a full extended Press-Schechter merger tree based on Parkinson, Cole, & Helly (2008), and then evolve the baryonic components along the merger-tree branches, employing analytical prescriptions at the “nodes” of the tree to mimic the effects of the mergers, and taking into account environmental effects (tidal stripping, tidal evaporation and dynamical friction) using the results of Taffoni et al. (2003).

The MBH are evolved within this model starting from two possible scenarios for their seeds, namely a light-seed ($M_{\text{seed}} \sim 150 M_{\odot}$) scenario – in which the MBHs form as remnants of Pop III stars at $z \sim 20$ (Madau & Rees 2001) – and a heavy-seed ($M_{\text{seed}} \sim 10^5 M_{\odot}$) scenario – in which MBHs form from the collapse of massive protogalactic disks at $10 \lesssim z \lesssim 15$ (Koushiappas, Bullock, & Dekel 2004; Begelman, Volonteri, & Rees 2006; Lodato & Natarajan 2006). The MBHs are then evolved along the “branches” of the merger tree, using the “reservoir” model of Granato et al. (2004); Lapi et al. (2006) to describe the gas present in the nuclear region, and taking into account the feedback they exert on the growth of structure by means of accretion-powered jets. As for the

accretion mechanism, we follow Dotti et al. (2010b); Maio et al. (2012) and assume that the nuclear cold gas accretes coherently (i.e., with a fixed angular momentum direction) onto the MBH in a gas-rich environment, thus resulting in a steady increase of the MBH spin (Bardeen 1970; Thorne 1974). In a gas-poor environment, due to the absence of a rotationally supported structure, we assume that the MBH accretes chaotically (i.e. in lumps of material with essentially random orientations of the orbital angular momentum), which results on average in a decrease of its spin (King & Pringle 2006). Similarly, prior to a MBH merger, we assume that the MBH spins are aligned due to the gravito-magnetic torques exerted by the circumbinary disk if the nuclear environment is gas rich (“wet merger”), whereas we assume that they are randomly oriented in a gas-poor nuclear environment (“dry merger”) (Bardeen & Petterson 1975; Bogdanović, Reynolds, & Miller 2007; Perego et al. 2009; Dotti et al. 2010a; Maio et al. 2012).

We stress this idealized picture will be at least partially modified by processes such as star formation and feedback, which can inject energy and angular momentum into the circumnuclear disk and potentially break the coherence of the accretion flow. However, recent simulations by Maio et al. (2012) show that these effects are probably not sufficient to create strongly turbulent environments and completely randomize the orbits of the accreting gas in the case of quasars fed by galaxy mergers, and that in these systems the accretion flow retains, at least partially, its coherence. The situation could be different for quasars fed by disk instabilities at high redshifts, where cold flows are important and accretion may take place by massive clumps (e.g. Bournaud et al. (2011); Dubois et al. (2011)). However, these clumps are expected to infall from large-scale disks, and might retain at least partially the disk’s angular momentum down to the MBH. In fact, observational hints for at least partial alignment come from the results of Lagos et al. (2011) (see also Lagos, Padilla, & Cora (2009)), who studied the relative orientation between AGNs and their host galaxies in the Sloan Digital Sky Survey, finding that random orientations are ruled out at high confidence.

The effect of MBH mergers is accounted for by using analytical formulas reproducing the results of numerical-relativity simulations. In fact, while the latter are the only way to study black-hole mergers rigorously, these simulations are very time-expensive and are not a viable option to cover the whole parameter space of black-hole binaries. Recently, the interface between numerical and analytical relativity has produced a number of approaches which employ a combination of post-Newtonian theory, general-relativistic perturbation theory and/or fits to numerical data to reproduce different aspects of black-hole binaries, such as the gravitational waveforms (see for instance the effective-one-body approach, e.g. Buonanno & Damour (1999); Damour, Jaranowski, & Schäfer (2008); Barausse & Buonanno (2010); Pan et al. (2011), or the hybrid waveforms, e.g. Santamaría et al. (2010)), the kick velocity (Baker et al. 2007, 2008; van Meter et al. 2010; Campanelli et al. 2007; Lousto & Zlochower 2009, 2011), the final mass (Tichy & Marronetti 2008; Lousto et al. 2010; Reisswig et al. 2009; Kesden 2008) and the final spin (Tichy & Marronetti 2008; Lousto et al. 2010; Rezzolla et al. 2008; Barausse & Rezzolla 2009; Buonanno, Kidder, & Lehner 2008). For this investigation, we use the formula of Barausse & Rezzolla (2009) for the spin, the formulas of Tichy & Marronetti (2008); Reisswig et al. (2009) for the mass, and the formula of van Meter et al. (2010) for the kick velocity.

Because we track the evolution of baryonic structures,

and in particular of the gas present in galactic nuclei, along the dark-matter merger trees, we can attempt to discriminate between chaotic or coherent accretion onto the MBHs, and between aligned-spin and precessing-spin MBH mergers. This improves upon previous models, e.g. Berti & Volonteri (2008); Volonteri et al. (2005), Fanidakis et al. (2010, 2011) and Lagos, Padilla, & Cora (2009), which considered *either* chaotic *or* coherent accretion, and *either* aligned *or* misaligned mergers.¹ We stress that LISA/SGO or eLISA/NGO will be able to test our model not only by measuring the MBH masses and spins as a function of redshift (Sesana, Volonteri, & Haardt 2007; Sesana et al. 2011), but for each merger event it should also be able to determine whether the MBH binary has precessing or aligned spins, by looking at the higher order harmonics of the gravitational waveforms (Lang & Hughes 2006, 2007, 2008; Lang, Hughes, & Cornish 2011).

This paper is organized as follows. In Sec. 2 we present our semianalytical galaxy-formation model in detail, focusing on dark matter in Sec. 2.1, on the baryonic components in Sec. 2.2, and on merger and environmental effects in Sec. 2.3. In Sec. 3 we calibrate the free parameters of our model to reproduce existing observations at $z = 0$ and at $z > 0$. In Sec. 4 we present our predictions for the character of MBH mergers (i.e. whether they involve aligned or precessing spins), while in Sec. 5 we study the evolution of the spins of MBHs as a function of redshift. In Sec. 6 we draw our conclusions and present plans for future work.

Throughout this paper, we assume a flat Λ CDM cosmology with $\Omega_{\text{DM}} = 0.227$, $\Omega_b = 0.0456$, $H_0 = 70.4 \text{ km}/(\text{s Mpc})$ and $\sigma_8 = 0.809$ (Jarosik et al. 2011; Larson et al. 2011; Komatsu et al. 2011).

2 PHYSICAL MODEL

2.1 The dark-matter merger trees

For the dark-matter merger and accretion evolution, we adopt the extended Press-Schechter formalism of Cole et al. (2000), as modified by Parkinson, Cole, & Helly (2008). This algorithm reproduces the statistical properties of the dark-matter merger trees produced with cosmological numerical N-body simulations (Springel et al. 2005; Cole et al. 2008). More specifically, we start our merger trees at an initial redshift $z = 20$ in the case of a light-seed MBH scenario (in which MBHs form as remnants of Pop III stars (Madau & Rees 2001)), while in the case of a heavy-seed scenario, in which MBHs form from the collapse of massive protogalactic disks (Koushiappas, Bullock, & Dekel 2004; Begelman, Volonteri, & Rees 2006; Lodato & Natarajan 2006), we start our merger trees at $z = 15$. In the light-seed scenario, we assume that a halo of total mass M_{vir} forming at $15 < z \leq 20$ contains a black-hole seed with mass $M_{\text{seed}} = 150 M_{\odot}$ if $M_{\text{vir}} > 1.1 \times 10^7 h^{-1} M_{\odot}$. This corresponds to populating halos collapsing from the large- σ peaks of the primordial density field (Madau & Rees 2001; Volonteri, Haardt, & Madau

¹ Indeed, Berti & Volonteri (2008); Volonteri et al. (2005) did not describe the baryonic components and therefore did not try to distinguish between gas-poor and gas-rich nuclear environments. On the contrary, Fanidakis et al. (2010, 2011) and Lagos, Padilla, & Cora (2009) modelled the baryonic components in detail, but did not try to infer whether accretion is chaotic or coherent. Also, Fanidakis et al. (2010, 2011) always considered randomly oriented spins prior to mergers.

2003).² In the heavy-seed scenario, instead, we place black-hole seeds with mass $M_{\text{seed}} = 10^5 M_\odot$ in halos forming at $10 < z \leq 15$ with virial mass (Koushiappas, Bullock, & Dekel 2004; Sesana, Volonteri, & Haardt 2007)

$$M_{\text{vir}} > 10^7 \left(\frac{1+z}{18} \right)^{-3/2} \left(\frac{\lambda}{0.04} \right)^{-3/2} M_\odot, \quad (1)$$

where λ is the halo spin parameter that we will introduce shortly. In both scenarios, because little is known about the spin of the seeds, we choose the spin parameter $a_{\text{bh}} = cJ_{\text{bh}}/(GM_{\text{bh}}^2)$ of each black-hole seed randomly from a uniform distribution $-1 \leq a_{\text{bh}} \leq 1$. We stress, however, that the predictions of our model, and in particular those regarding the spins of MBHs, are qualitatively independent of this assumption as long as one looks at MBHs of mass $M_{\text{bh}} \gtrsim 3M_{\text{seed}}$. This is because a black hole loses memory of its initial spin when it accretes a mass comparable to its own (i.e., if a black hole of mass M_{bh} accretes coherently, which as we will show is the case at high redshifts, its spin becomes maximal after accreting a mass $\lesssim 2M_{\text{bh}}$ (Bardeen 1970)).

Also, when it forms, a halo of total mass M_{vir} is assumed to contain unprocessed hot gas (see Sec. 2.2) with mass $M_{\text{hot}} = f_{\text{coll}} M_{\text{vir}}$, where the baryonic collapse fraction f_{coll} is given by

$$f_{\text{coll}}(M_{\text{vir}}, z) = \frac{f_b}{(1 + 0.26M_f(z)/M_{\text{vir}})^3}, \quad (2)$$

with $f_b = \Omega_b/\Omega_m \approx 0.16$ (with $\Omega_m = \Omega_b + \Omega_{\text{DM}}$). The filtering mass as a function of redshift, $M_f(z)$, accounts for the effect of the ionizing extragalactic UV background produced by massive stars and quasars, which is able to partially reduce the baryonic content in low-mass systems (Gnedin 2000). In particular, we calculate $M_f(z)$ using the equations in Appendix B of Kravtsov, Gnedin, & Klypin (2004), assuming $z_{\text{overlap}} = 11$ and $z_{\text{reion}} = 10$ (Larson et al. 2011) (z_{overlap} and z_{reion} respectively correspond to the redshift at which multiple HII regions overlap, and to the redshift at which most of the medium is ionized). After the initial formation redshift, additional hot gas is brought in from the IGM by the dark matter that accretes onto the halo, and we therefore assume

$$\dot{M}_{\text{inf}} = f_{\text{coll}} \dot{M}_{\text{vir}}. \quad (3)$$

where the baryonic collapse fraction f_{coll} is given again, as a function of redshift, by Eq. (2).

The resolution ΔM of the merger trees (i.e., the mass scale below which matter is assumed to accrete on an existing halo rather than give rise to a merger) is chosen to keep the computational time to acceptable levels while following the dark-matter halos (and the baryonic components within them) to very high redshifts. In particular, we set $\Delta M = \min(10^{-3}M_0, 10^{10}M_\odot) \times (1+z)^{-3.5}$, M_0 being the final mass of the halo at $z = 0$. The value of ΔM at $z = 0$ is comparable or smaller than the resolutions typically used by semianalytical galaxy formation models (see e.g. Cole et al. (2000); Somerville et al. (2008)), and the redshift dependence is introduced following Volonteri, Haardt, & Madau (2003) in order to track the merger tree to high redshifts. However, because our simulations are computationally rather expensive (especially for large virial masses), in order to further cap the computational time while ensuring a range of masses that is sufficient to allow both minor

and major mergers at all redshifts, at each redshift step we also stop following the branches that have mass smaller than $\delta \times M_{\text{max}}(z)$, where $\delta = 0.01$ and $M_{\text{max}}(z)$ is the mass of the most massive halo at that redshift.

The virial radius r_{vir} of a halo is related to its mass M_{vir} by the standard relation $M_{\text{vir}} = 4\pi r_{\text{vir}}^3 \rho_{\text{crit}} \Delta_c/3$, where ρ_{crit} is the critical density, and where the density contrast at virialization, Δ_c , is calculated following Bryan & Norman (1998). The halo density is assumed to be described by the fitting function of Navarro, Frenk, & White (1997) (NFW)

$$\rho_{\text{NFW}}(r) = \rho_s \left(\frac{r}{r_s} \right)^{-1} \left(1 + \frac{r}{r_s} \right)^{-2}, \quad (4)$$

truncated at the virial radius of the halo. The scale radius r_s of the NFW profile is related to the virial radius by the so-called concentration parameter $c(z) \equiv r_{\text{vir}}/r_s$. Imposing that the total halo mass equal M_{vir} , one immediately obtains that the scale density ρ_s is given by $\rho_s = M_{\text{vir}}(z)/[4\pi r_s^3 f(c)]$, with

$$f(c) = \ln(1+c) - \frac{c}{1+c}. \quad (5)$$

The concentration parameter has been studied by several authors (Bullock et al. 2001; Wechsler et al. 2002; Zhao et al. 2003a,b; Macciò et al. 2007), and found to present a large scatter for a fixed halo mass, but to scale generally with the halo's main-progenitor history. Following Bullock et al. (2001) and Wechsler et al. (2002), we adopt here a scaling $c(z) \propto 1/(1+z)$ for the main-progenitor history after the halo formation:

$$c_{\text{MPH}}(z) = \max \left(\frac{c_0}{1+z}, c_f \right), \quad (6)$$

where the concentration at formation is $c_f = 4.1$ (Wechsler et al. 2002) and the concentration at $z = 0$ is set following Macciò et al. (2007):

$$\log_{10} c_0 = 1.071 - 0.098 \left[\log_{10} \left(\frac{M_{\text{vir},0}}{M_\odot} \right) - 12 \right]. \quad (7)$$

The concentration of halos not belonging to the main-progenitor history, however, is not expected to be given by Eq. (6). Indeed, smaller halos are expected to be more concentrated (cf. Zhao et al. (2003a,b), as well as the environmental effects described by Bullock et al. (2001)). To account for this effect, we adopt the high-concentration limit of the expressions of Zhao et al. (2003a), which give $c \propto M_{\text{vir}}^{(\alpha-1)/(3\alpha)}$ (with $\alpha = 0.48$) at a fixed redshift. Combining this scaling with Eq. (6) we obtain the expression for the concentration of a halo of mass $M_{\text{vir}}(z)$ at redshift z :

$$c(z, M_{\text{vir}}(z)) = \max \left[\frac{c_0}{1+z} \left(\frac{M_{\text{vir}}(z)}{M_{\text{MPH}}(z)} \right)^{(\alpha-1)/(3\alpha)}, c_f \right]. \quad (8)$$

For simplicity, in this expression we assume that the main-progenitor history is given by $M_{\text{MPH}}(z) = M_{\text{vir},0} \exp(-S a_f z)$, with $S = 2$ and $a_f = c_f/c_0$ (Wechsler et al. 2002).³

The angular momentum of each halo is determined by the halo's *spin parameter*, defined as $\lambda = J_{\text{vir}} E_{\text{vir}}^{1/2} M_{\text{vir}}^{-5/2} G^{-1}$, where E_{vir} and J_{vir} are the total energy and angular momentum

² In our cosmology, a mass of $1.1 \times 10^7 h^{-1} M_\odot$ corresponds to the cosmological Jeans mass collapsing at $z = 20$ from the 3.5σ -peaks of the primordial density field (Volonteri, Haardt, & Madau 2003).

³ Applying Eq. (8) using the main-progenitor history extracted from the merger tree under consideration would be more difficult to implement in our code, and it is not clear that this procedure would be more accurate than the simple one that we use in this paper.

of the halo. We assign a spin parameter to a halo with no progenitors drawing from a log-normal distribution with median value $\bar{\lambda} = 0.039$ and standard deviation $\sigma = \sqrt{\langle (\ln \lambda - \ln \bar{\lambda})^2 \rangle} = 0.53$ (Cole & Lacey 1996; Cole et al. 2000). We then assume that this spin parameter remains unchanged along the cosmic history of the halo, except if it experiences a merger with a second halo of comparable mass (i.e. if $M_{\text{vir}2}/M_{\text{vir}1} > 0.3$), in which case we randomize the spin of the resulting halo by drawing from the same log-normal distribution. (We will discuss this in more detail in Sec. 2.3).

2.2 The baryonic matter

2.2.1 The hot gas phase

We assume that the hot unprocessed gas phase is isothermal at the halo's virial temperature T_{vir} , and in hydrostatic equilibrium within the NFW profile, such that

$$\rho_{\text{hot}}(r) = \rho_0 \exp \left[-\frac{27}{2} \beta \left\{ 1 - \frac{\ln(1 + r/r_s)}{r/r_s} \right\} \right], \quad (9)$$

with

$$\beta = \frac{8\pi\mu m_p G \rho_s r_s^2}{27k_B T_{\text{vir}}}, \quad (10)$$

where m_p is the proton mass, μ is the mean molecular mass and ρ_0 is calculated by normalizing to the total hot-gas mass at the redshift under consideration. The hot gas cools on a timescale t_{cool} given, at each point of its density distribution, by the standard expression

$$\tau_{\text{cool}}(r) = \frac{3\rho_{\text{hot}}(r)k_B T_{\text{vir}}}{2\mu m_p n_e(r)^2 \Lambda(T_{\text{vir}}, Z)}, \quad (11)$$

where $n_e(r)$ is the electron number density (which is proportional to $\rho_{\text{hot}}(r)$), $\Lambda(T, Z)$ is the cooling function given by the tabulated results of Sutherland & Dopita (1993), and where we assume that the hot gas is unprocessed and therefore has primordial metallicity $Z = 10^{-3} Z_{\odot}$.

If the cooling time of the hot gas is shorter than its free-fall time, given approximately, at each radius, by

$$t_{\text{dyn}}(r) = \sqrt{\frac{3\pi}{32G\bar{\rho}_{\text{NFW}}(r)}} \quad (12)$$

(where $\bar{\rho}_{\text{NFW}}(r)$ is the NFW average density within a radius r), then the hot-gas phase cools “fast” and undergoes gravitational collapse. If instead the cooling time is longer than the free-fall time, then the cooling is “slow” and the hot gas cools down through a sequence of quasi-hydrostatic equilibrium states. We therefore assume that the hot-gas phase is transferred into a cold-gas phase with rate

$$\dot{M}_{\text{coll}}(z) = 4\pi \int_0^{r_{\text{vir}}(z)} \frac{r^2 \rho_{\text{hot}}(r, z)}{t_{\text{coll}}(r, z)} dr, \quad (13)$$

where $t_{\text{coll}}(r, z) = \max(t_{\text{cool}}(r, z), t_{\text{dyn}}(r, z))$.

However, on top of this “classical” picture employed already by early semianalytical galaxy-formation models (see e.g. Kauffmann, White, & Guiderdoni (1993); Cole et al. (1994, 2000); Somerville & Primack (1999)), we also include the effects of cold accretion flows (Dekel & Birnboim 2006; Dekel et al. 2009; Cattaneo et al. 2006), which have been shown to be the predominant mechanism leading to the formation of low-mass systems. In halos with mass lower than the critical mass

$$M_c = M_{\text{shock}} \max[1, 10^{1.3(z-z_c)}], \quad (14)$$

where $M_{\text{shock}} = 2 \times 10^{12} M_{\odot}$ and $z_c = 3.2$, we assume that all the gas accreted from the IGM is *not* shock heated to the halo's virial temperature, but streams in on a dynamical time (Dekel & Birnboim 2006; Dekel et al. 2009; Cattaneo et al. 2006), thus enhancing star formation at high redshifts relative to the scenario where the accreting gas is shock heated. From the point of view of our model, this is equivalent to assuming directly $t_{\text{coll}} = t_{\text{dyn}}$ in (13), for halos with $M_{\text{vir}} < M_c$. Finally, in order to mimic the effect of ram pressure (Book & Benson 2010) and clumpy accretion (Dekel & Birnboim 2008; Birnboim & Dekel 2011), which are expected to quench the cooling of the hot gas at low redshifts in large halos, we set t_{coll} to the Hubble time in halos with $M_{\text{vir}}(z) > 10^{13} M_{\odot}$ and $z < 2$.

2.2.2 Density profile of the baryonic structures

For the growth of the baryonic structures, we here adopt the widely accepted scenario in which the hot-gas collapse gives rise to a disk of cold gas with mass $M_{d,\text{gas}}$, where star formation may occur and result in the formation of a stellar disk with mass $M_{d,\text{stars}}$. These disks may be disrupted by major galactic mergers and bar instabilities, thus forming a gaseous bulge with mass $M_{b,\text{gas}}$ and a stellar bulge with mass $M_{b,\text{stars}}$. Stellar formation takes place in the gaseous bulge as well, further contributing to $M_{b,\text{stars}}$. Also, as we will explain below, we allow part of the gaseous bulge to flow into a disk-like reservoir with mass M_{res} , which feeds the central MBHs during the accretion phase, and which forms the circumbinary disk expected to surround black-hole binaries after galactic mergers.

Assuming a dissipationless collapse of the hot gas into the gaseous disk, the angular momentum conservation allows one to relate the halo's virial radius and spin parameter to the radius of the disk. In particular, adopting an exponential surface-density profile for the gaseous disk,

$$\Sigma_d(r, z) = \Sigma_0(z) e^{-r/r_d^{\text{gas}}(z)}, \quad (15)$$

the scale radius is given by (Mo, Mao, & White 1998)

$$r_d^{\text{gas}}(z) = \frac{\lambda}{\sqrt{2}} \frac{j_d}{m_d} \frac{r_{\text{vir}}(z)}{\sqrt{f_c}} f_r(\lambda, c, m_d, j_d), \quad (16)$$

with

$$f_c = \frac{c}{2} \frac{1 - 1/(1+c)^2 - 2\ln(1+c)/(1+c)}{[c/(1+c) - \ln(1+c)]^2} \quad (17)$$

and

$$f_r(\lambda, c, m_d, j_d) = 2 \left[\int_0^\infty e^{-u} u^2 \frac{V_c(r_d u)}{V_c(r_{\text{vir}})} du \right]^{-1}, \quad (18)$$

where $V_c(r)$ is the velocity profile of the composite system (bulges, reservoir, disks, hot gas and dark matter) and where m_d and j_d are the ratios between the total mass and angular momentum of the disk and those of the halo. More specifically, we take $m_d = (M_d^{\text{stars}} + M_d^{\text{gas}})/M_{\text{vir}}$ and assume $j_d = m_d$ (Mo, Mao, & White 1998).

In the calculation of the disk's scale radius, we also account for the adiabatic halo response, which affects the velocity profile V_c entering Eq. (18), using the standard prescription of Blumenthal et al. (1986). In particular, the angular momentum conservation implies that the halo contraction following the baryonic collapse is described by

$$M_i(r_i)r_i = M_f(r_f)r_f, \quad (19)$$

where r_i and r_f are the initial and final radius of the shell under

consideration; $M_i(r_i)$ is the mass of the dark matter and hot gas contained in a radius r_i , calculated with the initial mass distribution (which is assumed to be given by the NFW density profile for both the dark matter and the hot gas); and $M_f(r_f)$ is the mass of the composite system (dark matter, hot gas, disks, bulges and reservoir) contained in the radius r_f . Moreover, the mass conservation ensures

$$M_f(r_f) = M_d(r_f) + M_b(r_f) + M_{\text{res}}(r_f) + M_{\text{hot}}(r_f) + M_{\text{DM}}(r_f) = M_d(r_f) + M_b(r_f) + M_{\text{res}}(r_f) + (1 - f_{\text{gal}})M_i(r_i), \quad (20)$$

where $M_X(r)$ is the mass of component X within a radius r , and where $f_{\text{gal}} = M_{\text{gal}}/M_{\text{vir}}$ (with $M_{\text{gal}} = M_d + M_b + M_{\text{res}}$, M_d and M_b being the total, i.e. gaseous and stellar, disk and bulge masses). Assuming spherical collapse without shell crossing, we can adopt the ansatz $r_f = \Gamma r_i$, where $\Gamma = \text{const}$ is the contraction factor (Blumenthal et al. 1986). Eqs. (19) and (20) can then be solved numerically for Γ , which then allows one to include the effect of the adiabatic contraction following the baryon collapse in the calculation of the velocity profile V_c of the composite system.

For the stellar disk, we assume an exponential surface-density profile with scale radius $r_d^{\text{stars}} = r_d^{\text{gas}}/2$ following Somerville et al. (2008) and Zavala et al. (2003) (see also Dutton & van den Bosch (2009), who show that the gaseous disk is theoretically expected to have a larger scale radius than the stellar disk). As for the gaseous and stellar bulges, we assume that they are described by the Hernquist profile (Hernquist 1990)

$$\rho_b^*(r) = \frac{M_b^*}{2\pi} \frac{r_b}{r(r+r_b)^3}, \quad * = \text{stars, gas}, \quad (21)$$

where the scale radius r_b (which we assume to be the same for the stellar and gaseous components) is related to the half-light radius R_{eff} by $r_b = R_{\text{eff}}/1.8153$ (Hernquist 1990). Using the fit of Shen et al. (2003) for R_{eff} ,

$$\log_{10}(R_{\text{eff}}) = \begin{cases} -5.54 + 0.56 \log_{10} \left(\frac{M_b}{M_{\odot}} \right) & \text{for } \log_{10} \left(\frac{M_b}{M_{\odot}} \right) > 10.3 \\ -1.21 + 0.14 \log_{10} \left(\frac{M_b}{M_{\odot}} \right) & \text{for } \log_{10} \left(\frac{M_b}{M_{\odot}} \right) \leq 10.3 \end{cases}$$

we can express the scale radius in terms of the total bulge mass $M_b = M_{b,\text{stars}} + M_{b,\text{gas}}$.

For simplicity⁴, we also assume that the reservoir can be described by an exponential surface-density profile with scale radius r_{res} proportional to the influence radius of the central MBH, i.e. $r_{\text{res}} = \alpha G M_{\text{bh}} / V_{\text{vir}}^2$, with $\alpha \approx 100$.

2.2.3 Star formation, supernova feedback and disk instabilities

Assuming that star formation in galactic disks only happens inside dense molecular clouds, which are well traced by the HCN luminosity (Wu et al. 2005; Gao & Solomon 2004), we follow

⁴ The reservoir's geometry is needed for instance to calculate the velocity V_c of the composite system, entering e.g. in Eq. (18), in the calculation of the adiabatic halo contraction factor Γ [Eqs. (19) and (20)], and in that the total gravitational potential ϕ appearing in Eqs. (27) and (30). However, the specific choice of the reservoir's geometry does not impact our final results significantly, because of its small size relative to the other components.

Blitz & Rosolowsky (2006); Dutton & van den Bosch (2009) and take the disk star-formation rate (SFR) to be proportional to the molecular-cloud surface density as traced by HCN, $\Sigma_{\text{mol,HCN}}$:

$$\dot{\Sigma}_{\text{sfr}} = \tilde{\epsilon}_{\text{sf}} \Sigma_{\text{mol,HCN}}, \quad (22)$$

where $\tilde{\epsilon}_{\text{sf}} = 13 \text{Gyr}^{-1}$. From this expression one easily obtains the total SFR by integrating over the surface of the gaseous disk.

More specifically, we write $\Sigma_{\text{mol,HCN}}$ as the product of the HCN fraction $R_{\text{HCN}} = \Sigma_{\text{mol,HCN}}/\Sigma_{\text{mol}}$ with the total molecular surface density of the disk, Σ_{mol} , which in turn we write as $\Sigma_{\text{mol}} = f_{\text{mol}} \Sigma_{\text{d,gas}}$, f_{mol} being the molecular fraction of the disk's gas. For R_{HCN} we use the fitting relation of Blitz & Rosolowsky (2006),

$$R_{\text{HCN}} = 0.1 \times (1 + \Sigma_{\text{mol}}/(200 M_{\odot} \text{pc}^{-2}))^{0.4}, \quad (23)$$

and we relate $f_{\text{mol}} = R_{\text{mol}}/(R_{\text{mol}} + 1)$ (with $R_{\text{mol}} = \Sigma_{\text{mol}}/\Sigma_{\text{atom}}$) to the mid-plane pressure P_{mp} of the disk, following again Blitz & Rosolowsky (2006):

$$R_{\text{mol}} = \left(\frac{P_{\text{mp}}/k_B}{4.3 \times 10^4} \right)^{0.92}, \quad (24)$$

where the pressure and the Boltzmann constant are in CGS units. For the mid-plane pressure of the disk, we assume (Elmegreen 1989; Blitz & Rosolowsky 2006; Dutton & van den Bosch 2009)

$$P_{\text{mp}} = \frac{\pi}{2} G \Sigma_g (\Sigma_g + (\sigma_g/\sigma_s) \Sigma_s), \quad (25)$$

where we use $\sigma_g/\sigma_s = 0.1$ (Dutton & van den Bosch 2009).

We stress that in high-mass (and thus high-density) galaxies, where the molecular fraction $f_{\text{mol}} \approx 1$, this star-formation prescription reduces to the standard Schmidt-Kennicutt star formation power law (Kennicutt 1998), $\dot{\Sigma}_{\text{sfr}} = \epsilon_{\text{sf}} [\Sigma_{\text{d,gas}}/(M_{\odot} \text{pc}^{-2})]^n$ with $n = 1.4$ and $\epsilon_{\text{sf}} = 1.6 \times 10^{-4} M_{\odot} \text{kpc}^{-2} \text{yr}^{-1}$, whereas in low-density systems the exponent n approaches 2.84. As a result, the star formation law that we adopt suppresses star formation in low-mass galaxies, in accordance with observations (see Dutton & van den Bosch (2009); Blitz & Rosolowsky (2006) for a detailed discussion).

The feedback from supernova events is expected to eject cold gas from the disk. To do so, the energy released by supernova explosions in the disk must be sufficient to unbind the cold gas. We therefore compare, at each radius, the amount of energy released by these explosions with the binding energy, and write the total amount of cold gas ejected from the system as

$$\dot{M}_{\text{SN}}^d(z) = 2\pi \int_0^{r_{\text{vir}}} r \dot{\Sigma}_{\text{SN}}(r, z) dr, \quad (26)$$

where

$$\dot{\Sigma}_{\text{SN}}(r, z) = - \frac{\epsilon_{\text{SN}} E_{\text{SN}} \eta_{\text{SN}} \dot{\Sigma}_{\text{sfr}}(r, z)}{\phi(r, z)}. \quad (27)$$

Here, $\phi(r, z)$ is the binding energy per unit mass of the composite system (bulges, disks, reservoir, hot gas and dark matter), η_{SN} is the number of Type II supernovae expected per solar mass of stars formed⁵, $E_{\text{SN}} = 10^{44} \text{J}$ is the kinetic energy released per supernova event, and ϵ_{SN} is a parameter ranging from 0 to 1 and regulating the efficiency with which the supernova energy is transferred to the cold gas. We stress that the supernova feedback is most effective

⁵ Following Romano et al. (2005), we adopt a Chabrier (2003) initial mass function (IMF) between $0.001 M_{\odot}$ and $1 M_{\odot}$, and a power law $dN/dm_{\star} \propto m_{\star}^{-2.7}$, therefore steeper than the standard Salpeter IMF, between $1 M_{\odot}$ and $100 M_{\odot}$. This IMF gives $\eta_{\text{SN}} = 5 \times 10^{-3} M_{\odot}^{-1}$.

in low-mass systems, which present shallower potential wells from which the cold gas can easily escape due to supernova explosions.

Also, disks are known to develop bar instabilities when they become self-gravitating, thus getting disrupted and transferring material to the bulge components (Christodoulou, Shlosman, & Tohline 1995; Efstathiou, Lake, & Negroponte 1982). We assume that a stellar or gaseous disk is stable if, respectively,

$$\frac{V_c(2.2r_d)}{(GM_d^*/r_d^*)^{1/2}} > \alpha_{\text{crit}}^* \quad * = \text{stars, gas}, \quad (28)$$

where $\alpha_{\text{crit}}^{\text{stars}} = 1.1$ and $\alpha_{\text{crit}}^{\text{gas}} = 0.9$ (Christodoulou, Shlosman, & Tohline 1995; Efstathiou, Lake, & Negroponte 1982). If the disk becomes unstable, we assume that it gets disrupted in a dynamical time and transfers its material (either stars or gas) to the corresponding bulge component.

As for the gaseous bulge, we assume that star formation takes place on a dynamical timescale. More specifically, we assume that the SFR per spherical shell in the gaseous bulge can be calculated as

$$\frac{d\psi_b(r, t)}{dr} = 4\pi r^2 \frac{\rho_{b,\text{gas}}(r)}{t_{\text{gas}}(r)}, \quad (29)$$

where $t_{\text{gas}}(r) = \sqrt{3\pi/(32G\rho_{b,\text{gas}})}$ is the local dynamical time for the bulge gas. Eq. (29) can then be integrated over all radii to give the total SFR in the bulge. As in the disk case, we assume that the star formation ejects cold gas as a result of supernova explosions, at a rate

$$\dot{M}_{b,\text{gas}}^{\text{SN}}(t) = - \int \frac{\epsilon_{\text{SN}} E_{\text{SN}} \eta_{\text{SN}} d\psi_b(r, t)/dr}{\phi(r, t)} dr, \quad (30)$$

where the efficiency ϵ_{SN} is assumed to be the same as for the disk. Again, this feedback mechanism is most effective in low-mass systems.

Finally we note that the SFR rates (22) and (29) do not account for the mass returned to the cold-gas phase by short-lived stars in the form of processed material. To include this effect, we use the instant-recycling approximation and assume that a fraction $R = 0.29$ of the mass is instantly returned into the cold-gas phase.⁶ In particular, this implies that the effective SFRs regulating the evolution of the disks and bulges are

$$\dot{M}_b^{\text{SFR}}(t) = (1 - R) \int \frac{\psi_b(r, t)}{dr} dr, \quad (31)$$

$$\dot{M}_d^{\text{SFR}}(t) = (1 - R) \int 2\dot{\Sigma}_{\text{sfr}}(r, t) \pi r dr. \quad (32)$$

2.2.4 The evolution of MBHs

Star formation in the bulge is believed to force, e.g. by radiation drag (Umemura 2001; Kawakatu & Umemura 2002; Kawakatu, Umemura, & Mori 2003), part of the bulge's cold gas onto a low angular momentum circumnuclear reservoir, which feeds the central MBHs during the accretion phases (Haiman, Ciotti, & Ostriker 2004), and which may be identified with the circumbinary disks expected to surround MBH binaries after a galactic merger. Because star formation in the bulge

happens in violent bursts triggered by disk instabilities (see previous section) or by galaxy mergers (as we will explain in the next section), we identify this accretion mode with the quasar mode of MBHs.

In particular, we assume that the growth rate of the reservoir is proportional to the bulge SFR (Granato et al. 2004; Lapi et al. 2006) and is given by

$$\dot{M}_{\text{res}} = A_{\text{res}} \psi_b(t), \quad (33)$$

where A_{res} is a free parameter of our model. The cold gas in this reservoir then becomes available to accrete onto the central MBH at a rate

$$\dot{M}_{\text{QSO}} = \frac{\dot{M}_{\text{res}}}{t_{\text{accr}}}, \quad (34)$$

where the timescale t_{accr} is a free parameter regulating the infall of the reservoir gas into the (pc-scale) MBH accretion disk. The accretion of this gas then changes the MBH mass according to

$$\dot{M}_{\text{bh,QSO}} = \dot{M}_{\text{QSO}}(1 - \eta(a_{\text{bh}})), \quad (35)$$

where the spin-dependent efficiency $\eta(a_{\text{bh}})$ measures the energy emitted in electromagnetic radiation by the accretion disk. More specifically, we follow Dotti et al. (2010b); Maio et al. (2012) and assume that the accretion onto the MBH takes place coherently (i.e. with a fixed angular momentum direction, cf. Bardeen (1970); Thorne (1974)) in a gas-rich environment, where gravito-magnetic torques rapidly align the the disk's angular momentum with the spin of the MBH.⁷ (As mentioned in the introduction, this idealized coherent accretion flow may be at least partially randomized by star formation and feedback effects in the circumnuclear disk, or by the formation of clumps in high-redshift disk galaxies.) In a gas-poor environment, due to the absence of a rotationally supported structure, we assume that the MBH accretes chaotically (Dotti et al. 2010b) (i.e. in lumps of material with essentially random orientations of the orbital angular momentum, cf. King & Pringle (2006)). Identifying a gas-rich environment with one where $\dot{M}_{\text{res}} > \dot{M}_{\text{bh}}$, we assume that the efficiency is

$$\eta(a_{\text{bh}}) = 1 - E_{\text{ISCO}}(a_{\text{bh}}) \quad (36)$$

for $\dot{M}_{\text{res}} > \dot{M}_{\text{bh}}$, with E_{ISCO} the specific energy at the anti-clockwise innermost stable circular orbit (ISCO) around a Kerr black hole with spin parameter a_{bh} ranging between -1 (extremal spin pointing downwards) and 1 (extremal spin pointing upwards) (Bardeen 1970). Identifying instead a gas-poor environment with one where $\dot{M}_{\text{res}} < \dot{M}_{\text{bh}}$, we assume

$$\eta(a_{\text{bh}}) = 1 - \frac{E_{\text{ISCO}}(a_{\text{bh}}) + E_{\text{ISCO}}(-a_{\text{bh}})}{2} \quad (37)$$

for $\dot{M}_{\text{res}} < \dot{M}_{\text{bh}}$, where we have assumed that accretion has equal probability of happening in the clockwise or anti-clockwise directions. (This is a simplified version of the prescription derived by King et al. (2005).)

Because of the energy and angular momentum carried by the gas accreting onto the MBH, in a gas-rich environment ($\dot{M}_{\text{res}} > \dot{M}_{\text{bh}}$) the spin parameter a_{bh} increases steadily under coherent accretion:

$$\dot{a}_{\text{bh,QSO}}^{\text{coherent}} = [L_{\text{ISCO}}(a_{\text{bh}}) - 2a_{\text{bh}}E_{\text{ISCO}}(a_{\text{bh}})] \frac{\dot{M}_{\text{QSO}}}{\dot{M}_{\text{bh}}}, \quad (38)$$

⁶ To calculate this return rate, we follow again Romano et al. (2005) and adopt a Chabrier (2003) IMF between $0.001M_{\odot}$ and $1M_{\odot}$, and a power law $dN/dm_{\star} \propto m_{\star}^{-2.7}$ between $1M_{\odot}$ and $100M_{\odot}$.

⁷ This is the so-called Bardeen-Petterson effect (Bardeen & Petterson 1975), which also plays a fundamental role during gas-rich MBH mergers, as we will explain shortly.

where again E_{ISCO} and L_{ISCO} are the specific energy and angular momentum at the anti-clockwise ISCO around a Kerr black hole with spin parameter a_{bh} ranging between -1 (extremal spin pointing downwards) and 1 (extremal spin pointing upwards). In a gas-poor environment ($M_{\text{res}} < M_{\text{bh}}$), we again assume that accretion can happen clockwise or anticlockwise with equal probabilities, and the spin parameter decreases (on average) under chaotic accretion, following

$$\dot{a}_{\text{bh,QSO}}^{\text{chaotic}} = \left\{ \frac{L_{\text{ISCO}}(a_{\text{bh}}) + L_{\text{ISCO}}(-a_{\text{bh}})}{2} - a_{\text{bh}}[E_{\text{ISCO}}(a_{\text{bh}}) + E_{\text{ISCO}}(-a_{\text{bh}})] \right\} \frac{\dot{M}_{\text{QSO}}}{M_{\text{bh}}} . \quad (39)$$

We stress that we do *not* restrict the MBH accretion rate to values lower than the Eddington rate, i.e. we allow super-Eddington mass accretion. However, following the theory of thin accretion disks (Shakura & Sunyaev (1973); see also Poutanen et al. (2007)), we assume the MBH's bolometric luminosity to be

$$L_{\text{bh,QSO}} = \min \left\{ \eta(a_{\text{bh}}) \dot{M}_{\text{QSO}} c^2, L_{\text{Edd}} \left[1 + \ln \left(\frac{\eta(a_{\text{bh}}) \dot{M}_{\text{QSO}} c^2}{L_{\text{Edd}}} \right) \right] \right\} . \quad (40)$$

Also, thin-disk accretion is believed to produce jet outflows due to the Blandford-Znajek effect (Blandford & Znajek 1977), and the jet power is parameterized by (Meier 2001)

$$L_{\text{jet,QSO}} \approx f_{\text{jet}} \times 10^{42.7} \text{erg s}^{-1} \left(\frac{\alpha}{0.01} \right)^{-0.1} m_9^{0.9} \left(\frac{\dot{m}}{0.1} \right)^{6/5} \times (1 + 1.1a_{\text{bh}} + 0.29a_{\text{bh}}^2), \quad (41)$$

where α is the disk's viscosity parameter (for which we assume $\alpha = 0.1$), $m_9 = M_{\text{bh}}/(10^9 M_{\odot})$, $\dot{m} = \dot{M}_{\text{QSO}}/(22 m_9 M_{\odot} \text{yr}^{-1})$, and where f_{jet} is a “fudge” factor parameterizing the uncertainties affecting Eq. (41) (e.g., the Blandford-Znajek jet-outflow power scales quadratically with the magnetic field, which is poorly known). These jets are expected to exert a feedback on the hot-gas phase and on the bulge gaseous component. More specifically, we assume that they eject hot gas and bulge cold gas from the system with rates (Granato et al. 2004)

$$\dot{M}_{\text{b,gas}}^{\text{QSO}} = \frac{2}{3} \frac{L_{\text{jet,QSO}}}{\sigma^2} \frac{M_{\text{b,gas}}}{M_{\text{hot}} + M_{\text{b,gas}}}, \quad (42)$$

$$\dot{M}_{\text{hot}}^{\text{QSO}} = \frac{2}{3} \frac{L_{\text{jet,QSO}}}{\sigma^2} \frac{M_{\text{hot}}}{M_{\text{hot}} + M_{\text{b,gas}}}. \quad (43)$$

with $\sigma = 0.65 V_{\text{vir}}$.

In addition to the thin-disk quasar accretion mode, MBHs are expected to quiescently accrete matter directly from the hot-gas phase, when that is in quasi-hydrostatic equilibrium, through a thick advection-dominated accretion flow (ADAF). This is usually dubbed “radio accretion mode” and does not contribute significantly to the mass evolution of MBHs, because the accretion rate is much smaller than for the quasar mode. However, the radio mode is believed to play an important role in galaxy formation because it produces jet outflows that are much more powerful than those that would be produced by a thin disk with the same accretion rate (Croton et al. 2006; Bower et al. 2006; Meier 2001). This compensates the smaller mass accretion rate of the radio mode relative to the quasar mode, and enhances its feedback. Moreover, while the quasar feedback is triggered by starbursts in the bulge, and therefore intermittent and important mostly at relatively high redshifts,

the radio-mode accretion and feedback are continuous and remains efficient up to $z = 0$.

We therefore assume that when $t_{\text{cool}} > t_{\text{ff}}$ (i.e., when the hot gas undergoes quasi-hydrostatic cooling, cf. Sec. 2.2.1) the MBHs accrete directly from the hot-gas phase at the Bondi accretion rate (Bondi 1952)

$$\dot{M}_{\text{bh,radio}} = 4\pi \lambda_B \rho_{\text{hot}} (G M_{\text{bh}})^2 / v_s^3, \quad (44)$$

where ρ_{hot} is the density of the hot gas in the center of the galaxy; v_s is the sound velocity in the hot gas, which is of the order of the virial velocity V_{vir} ; and λ_B is a constant that depends on the adiabatic index of the gas, with $\lambda_B \approx 1.12$ for an isothermal gas. The bolometric luminosity is then given (Mahadevan 1997) by the ADAF luminosity

$$L_{\text{bol,radio}} = 1.3 \times 10^{38} \left(\frac{M_{\text{bh}}}{M_{\odot}} \right) \left(\frac{\dot{m}}{\alpha^2} \right) \left(\frac{\beta}{0.5} \right) \text{erg s}^{-1}, \quad (45)$$

where $\dot{m} = \dot{M}_{\text{bh,radio}}/(22 m_9 M_{\odot} \text{yr}^{-1})$ and β is related to viscosity parameter $\alpha = 0.1$ by $\alpha \approx 0.55(1 - \beta)$ (Fanidakis et al. 2011). Because the radio-mode accretion happens through an ADAF and not through a thin disk, the rate of change of the spin parameter is different from the quasar-mode case. More specifically, if we assume that the accretion is geometrically spherical, no angular momentum is transferred to the black hole and the spin parameter decreases due to the mass increase:

$$\dot{a}_{\text{bh,radio}} = -2a_{\text{bh}} \frac{\dot{M}_{\text{bh,radio}}}{M_{\text{bh}}}. \quad (46)$$

Finally, ADAF accretion is expected to produce much more powerful jets than the quasar mode, and the jet power is parameterized by (Meier 2001)

$$L_{\text{jet}}^{\text{radio}} \approx f_{\text{jet}} \times 10^{45.1} \text{erg s}^{-1} \left(\frac{\alpha}{0.3} \right)^{-1} m_9 \left(\frac{\dot{m}}{0.1} \right) g^2 \times (0.55f^2 + 1.5fa_{\text{bh}} + a_{\text{bh}}^2), \quad (47)$$

where again $m_9 = M_{\text{bh}}/(10^9 M_{\odot})$ and $\dot{m} = \dot{M}_{\text{bh,radio}}/(22 m_9 M_{\odot} \text{yr}^{-1})$, and where f and g are dimensionless quantities, defined precisely in Meier (2001), regulating the actual angular velocity and azimuthal magnetic field of the system. Following Meier (2001) we set $f = 1$ and $g = 2.3$, but we include a “fudge” factor f_{jet} [assumed to be the same as in Eq. (41)] to account for the uncertainties in Eq. (47) (e.g. the uncertainties in the magnetic field, on which the jet power depends quadratically, and the higher-order terms in the black-hole spin, which are neglected in the standard Blandford-Znajek calculation, cf. Tanabe & Nagataki (2008); Tchekhovskoy, Narayan, & McKinney (2010)). Like in the case of the quasar mode, we assume that the jets remove hot gas and bulge cold gas from the system with rates (Granato et al. 2004)

$$\dot{M}_{\text{b,gas}}^{\text{radio}} = \frac{2}{3} \frac{L_{\text{jet,radio}}}{\sigma^2} \frac{M_{\text{b,gas}}}{M_{\text{hot}} + M_{\text{b,gas}}}, \quad (48)$$

$$\dot{M}_{\text{hot}}^{\text{radio}} = \frac{2}{3} \frac{L_{\text{jet,radio}}}{\sigma^2} \frac{M_{\text{hot}}}{M_{\text{hot}} + M_{\text{b,gas}}}. \quad (49)$$

Finally, we stress that while the supernova feedback is most effective in low-mass systems, which presents shallow potential wells, the quasar and radio-mode feedbacks are most important in massive galaxies, where the quasar and radio activity is most pronounced.

2.3 Mergers and environmental effects

The prescriptions outlined in the previous sections allow us to describe the evolution of the baryonic structures along the branches of the dark-matter merger trees. Therefore, in order to produce “baryonic merger trees” describing the complete evolution of the baryons along the cosmic history, we only need a prescription to describe what happens at the “nodes” of the dark-matter merger trees, i.e. when halos merge.

First of all, we should note that the “nodes” of the dark-matter merger trees correspond to the instant at which the smaller halo (the “satellite”) enters the bigger one (the “host”). After entering the host, the satellite halo survives as a bound substructure (a “subhalo”) within the host. However, because of the gravitational interaction with the particles of the host halo, known as dynamical friction, the satellite gradually loses energy and angular momentum and slowly sinks to the center of the host. It is only when the satellite halo reaches the center of the host that the subhalo finally dissolves and the baryonic structures of the two halos merge.

Because our dark-matter merger trees are not necessarily binary (i.e., their redshift step is such that a “node” may correspond to the merger of more than two halos), at each node we consider the biggest halo as the host, and calculate the dynamical-friction timescales for the remaining halos (the satellites) using the fitting formula proposed by Boylan-Kolchin, Ma, & Quataert (2008), which accurately reproduces the merger timescales of extended halos predicted by N-body simulations:

$$t_{\text{df}} = \frac{R_{\text{vir}}}{V_{\text{vir}}} A \frac{(M_{\text{host}}/M_{\text{sat}})^b}{\ln(1 + M_{\text{host}}/M_{\text{sat}})} \exp[c\epsilon] \left[\frac{r_c(E)}{R_{\text{vir}}} \right]^d, \quad (50)$$

with $A = 0.216$, $b = 1.3$, $c = 1.9$ and $d = 1$. Here, $\epsilon = L/L_c(E)$ and $r_c(E)$ are respectively the “orbital circularity” (the ratio between the satellite’s angular momentum L and the angular momentum L_c of a circular orbit with the same energy E as the satellite) and the “circular radius” (the radius of a circular orbit with the same energy E as the satellite). These quantities describe the initial conditions of the infall (i.e. on what orbit the satellite enters the host). In particular, ϵ may vary from 0 to 1 and is drawn from a normal distribution centered on $\bar{\epsilon} \approx 0.5$ and standard deviation $\sigma \approx 0.23$ (Tormen 1997; Khochfar & Burkert 2006), while r_c is derived from the periastron radius r_p , which N-body simulations suggest should be correlated with the circularity ϵ (more specifically, we assume $r_p = R_{\text{vir}}\epsilon^{2.17}$; cf. Tormen (1997); Khochfar & Burkert (2006)).

We notice that Eq. (50) already includes environmental effects such as the tidal stripping and tidal heating, which cause the mass of the satellite to decrease while sinking to the host’s center. This is because Eq. (50) fits the results of N-body simulations, where these effects are automatically included. However, Eq. (50) does not include the effect of the continuous accretion of dark matter and hot gas onto the host halo along cosmic time, and we therefore correct for this effect at each redshift step of our merger tree by rescaling the remaining dynamical friction time to account for the host having grown in the meantime.

Even though environmental effects are included in the dynamical friction timescale (50), as we have mentioned they also have the effect of reducing the satellite’s mass while it sinks in the host. In particular, following Taffoni et al. (2003), we account for the tidal stripping (i.e. the tidal truncation of the satellite’s density profile due to the average tidal force exerted by the host halo) by cutting the dark-matter and hot-gas density profiles of the satellite at the tidal radius R_{tidal} , corresponding to the distance, from the satel-

lite’s center, at which the mean density of the satellite is comparable to the host’s mean density at the satellite’s position r :

$$\bar{\rho}_{\text{sat}}^{\text{NFW}}(R_{\text{tidal}}) \approx \bar{\rho}_{\text{host}}^{\text{NFW}}(r). \quad (51)$$

More specifically, because the tidal stripping is most effective when the satellite is at the periastron of its orbit, we assume $r = r_p$ in this equation and perform the cut when the satellite first reaches the periastron. Besides the tidal stripping, the satellite also loses mass due to the tidal heating, i.e. the evaporation induced by the rapidly varying tidal forces near the periastron. We assume that this effect causes both the dark matter as well as all the baryon components to lose mass with characteristic timescale calculated as in the Appendix B of Taffoni et al. (2003), and we assume that this mass loss starts when the satellites first reaches the periastron.

When the satellite has sunk to the center of the host, a dynamical friction time t_{df} after it first entered the host, the satellite halo finally loses its identity, and the baryonic structures of the host and satellite merge as well. When such a merger happens, if the ratio $M_{\text{vir}}^{\text{sat}}/M_{\text{vir}}^{\text{host}}$ between the host and satellite halo masses is sufficiently large, we assume that the merger between the dark-matter structures perturbs the spin parameter of the resulting composite halo. More specifically, as already mentioned in Sec. 2.1, if $M_{\text{vir}}^{\text{sat}}/M_{\text{vir}}^{\text{host}} > 0.3$ we assign the final composite halo a new spin parameter λ drawn from the same distribution used for newly formed halos, i.e. a log-normal distribution with median value $\bar{\lambda} = 0.039$ and standard deviation $\sigma = \sqrt{(\ln \lambda - \ln \bar{\lambda})^2} > 0.53$ (Cole & Lacey 1996; Cole et al. 2000), while if $M_{\text{vir}}^{\text{sat}}/M_{\text{vir}}^{\text{host}} < 0.3$ we leave the host’s spin parameter unchanged.

Also, further complications to this picture arise at the nodes where two or more halos, some or all of which containing their own subhalos, meet. In this case, if $M_{\text{vir}}^{\text{sat}}/M_{\text{vir}}^{\text{host}} > 0.3$ for any one of the satellite halos, we recalculate all the dynamical friction timescales using Eq. (50), where we assume a new value for the circularity ϵ , drawn from the same distribution used when the satellites first entered the host, i.e. a normal distribution centered on $\bar{\epsilon} \approx 0.5$ and standard deviation $\sigma \approx 0.23$ (Tormen 1997; Khochfar & Burkert 2006). If instead $M_{\text{vir}}^{\text{sat}}/M_{\text{vir}}^{\text{host}} < 0.3$ for all the satellite halos, we calculate the dynamical friction times of the satellites and their subhalos in the host using the procedure that we just described, but leave the dynamical friction times of the subhalos of the host unchanged. This scenario corresponds to the intuitive picture in which the incoming satellites manage to perturb and randomize the orbits of the host’s subhalos only if they are sufficiently massive compared to the host.

We also recall that numerical simulations (Walker, Mihos, & Hernquist 1996; Naab & Burkert 2003; Bournaud, Jog, & Combes 2005) suggest that mergers where the mass ratio between the total baryonic masses is larger than $\sim 0.25 - 0.3$ (“major mergers”) disrupt the galactic disks and give rise to a spheroidal component, while “minor mergers” (i.e., with mass ratio between the total baryonic masses smaller than $\sim 0.25 - 0.3$) do not destroy the galactic disks, although they may perhaps drive the growth of the bulge by disk instabilities (see Sec. 2.2.3). We implement this scenario in our model by defining a merger as “major” if the ratio of the baryonic masses is larger than 0.25 ($M_{\text{baryon, sat}}/M_{\text{baryon, host}} > 0.25$ with $M_{\text{baryon}} = M_{\text{d, stars}} + M_{\text{d, gas}} + M_{\text{b, stars}} + M_{\text{b, gas}} + M_{\text{res}}$), otherwise we define the merger as minor. We then assume that in major mergers the disks are destroyed and their masses are added

to the corresponding bulge components

$$\begin{aligned}
 M_{d,\text{gas}} &= 0 \\
 M_{d,\text{stars}} &= 0 \\
 M_{b,\text{gas}} &= M_{b,\text{gas}}^{\text{host}} + M_{d,\text{gas}}^{\text{sat}} \\
 M_{b,\text{stars}} &= M_{b,\text{stars}}^{\text{host}} + M_{d,\text{stars}}^{\text{sat}} \\
 M_{\text{res}} &= M_{\text{res}}^{\text{host}} + M_{\text{res}}^{\text{sat}},
 \end{aligned} \tag{52}$$

while we assume that minor mergers do not affect the galactic morphology, and therefore simply add the satellite’s disk and bulge material to the disk and bulge component of the host galaxy:

$$\begin{aligned}
 M_{d,\text{gas}} &= M_{d,\text{gas}}^{\text{host}} + M_{d,\text{gas}}^{\text{sat}} \\
 M_{d,\text{stars}} &= M_{d,\text{stars}}^{\text{host}} + M_{d,\text{stars}}^{\text{sat}} \\
 M_{b,\text{stars}} &= M_{b,\text{stars}}^{\text{host}} + M_{b,\text{stars}}^{\text{sat}} \\
 M_{b,\text{gas}} &= M_{b,\text{gas}}^{\text{host}} + M_{b,\text{gas}}^{\text{sat}} \\
 M_{\text{res}} &= M_{\text{res}}^{\text{host}} + M_{\text{res}}^{\text{sat}}.
 \end{aligned} \tag{53}$$

Another effect of mergers is to cause significant starburst events in the merging galaxies. This is automatically accounted for in our model, because the disruption of the galactic disks following a major merger channels gas into the spheroidal component, where star formation is very efficient, because it happens on a dynamical timescale [cf. Eq. (29)].

2.3.1 Black-hole mergers

When the baryonic structures of the host and satellite merge, a dynamical friction time t_{df} after the satellite first entered the host, the central MBHs do not coalesce right away, but form a binary system. The binary then continues to harden through “slingshot” interactions (Saslaw, Valtonen, & Aarseth 1974), in which stars intersecting the binary are ejected at velocities comparable to the binary’s orbital velocity, thus increasing the binding energy of the binary system. However, the binary will soon eject all the intersecting stars, thus making the slingshot hardening inefficient. This will cause the binary to stall at a separation of ~ 1 pc, unless other mechanisms intervene to make it decay to a separation of ~ 0.01 pc, where gravitational-wave emission becomes strong enough to drive the binary’s evolution to the merger in a timescale shorter than the Hubble time. Since there is not, at present, a generally accepted scenario to overcome the stalling of the binary’s evolution at ~ 1 pc, this bottleneck is generally referred to as “the final-parsec problem” (Merritt & Milosavljević 2005). It is generally thought, however, that the final-parsec problem is somehow solved in nature, because uncoalesced binaries would result in slingshot ejection of MBHs when additional MBHs are brought in by successive mergers, thus resulting in off-center MBHs that seem rare or non-existent and in too much scatter in the $M - \sigma$ relation (Haehnelt & Kauffmann 2002). Also, some possible mechanisms that would harden the binary until gravitational-wave emission becomes important, possibly solving the final-parsec problem, have been proposed. For instance, the presence of a gaseous accretion disk would harden the binary on the viscosity timescale (Armitage & Natarajan 2002); or the supply of stars available for slingshot interaction may be replenished by star-star encounters (Milosavljević & Merritt 2003; Yu 2002), or as a result of the triaxial gravitational potential that one naturally expects in galaxies forming from major mergers (Merritt & Poon 2004; Berczik et al. 2006; Khan, Just, & Merritt 2011; Preto et al. 2011).

Because of all these uncertainties, and because the MBH coalescence timescale is in any case expected to be small compared to the dynamical friction time t_{df} and its uncertainties⁸, we make the simplifying assumption that the MBHs merge at the same time as the baryonic structures, i.e. a dynamical friction time t_{df} after the satellite first entered the host. This approximation is adopted, to our knowledge, by virtually all semianalytical galaxy-formation models proposed so far, and has the advantage of greatly simplifying the implementation of our model.⁹

When the two MBHs merge, we can determine the mass and spin of the resulting MBH remnant if we make some assumptions on the relative orientation of the spins at large separations. As mentioned in the introduction, in the last few years numerical relativity has reached a level of maturity sufficient for simulating black-hole binaries with non-aligned spins in a vast region of the space of parameters. Phenomenological formulas (Tichy & Marronetti 2008; Lousto et al. 2010; Rezzolla et al. 2008; Barausse & Rezzolla 2009; Reisswig et al. 2009; Buonanno, Kidder, & Lehner 2008; Kesden 2008) based on Post-Newtonian theory, general-relativistic perturbation theory, symmetry arguments, as well as fits to the numerical-relativity results, are not only capable of reproducing to high accuracy the simulation results for observables like the final mass, the final-spin magnitude and orientation, and the recoil velocity, but also allow one to make sensible predictions for these quantities in regions of the parameter space where simulations are still too time-expensive to ensure complete coverage (see Rezzolla (2009) for a review).

Here, we use the formula of Barausse & Rezzolla (2009), which predicts the MBH remnant’s spin magnitude and orientation to very high accuracy (see Barausse & Rezzolla (2009); Kesden, Sperhake, & Berti (2010)). In particular, the final-spin magnitude is

$$|\mathbf{a}_{\text{fin}}| = \frac{1}{(1+q)^2} \left[|\mathbf{a}_1|^2 + |\mathbf{a}_2|^2 q^4 + 2|\mathbf{a}_2||\mathbf{a}_1|q^2 \cos \alpha + 2(|\mathbf{a}_1| \cos \beta + |\mathbf{a}_2|q^2 \cos \gamma) |\ell|q + |\ell|^2 q^2 \right]^{1/2}, \tag{54}$$

with

$$\begin{aligned}
 |\ell| &= 2\sqrt{3} + t_2\nu + t_3\nu^2 + \\
 &\frac{s_4}{(1+q)^2} (|\mathbf{a}_1|^2 + |\mathbf{a}_2|^2 q^4 + 2|\mathbf{a}_1||\mathbf{a}_2|q^2 \cos \alpha) + \\
 &\left(\frac{s_5\nu + t_0 + 2}{1+q^2} \right) (|\mathbf{a}_1| \cos \beta + |\mathbf{a}_2|q^2 \cos \gamma).
 \end{aligned} \tag{55}$$

Here, $q = M_{\text{bh},2}/M_{\text{bh},1}$ is the mass ratio; $\nu = q/(1+q)^2$ is the symmetric mass ratio; $|\mathbf{a}_1|$ and $|\mathbf{a}_2|$ are the initial spin magnitudes; α, β, γ are the angles (at large separation) respectively between the two spins and between spin 1 (spin 2) and the direction of orbital angular momentum, $\hat{\mathbf{L}}$; and $s_4 = -0.1229 \pm 0.0075$, $s_5 = 0.4537 \pm 0.1463$, $t_0 = -2.8904 \pm 0.0359$, $t_3 = 2.5763 \pm 0.4833$ and $t_2 = -3.5171 \pm 0.1208$.¹⁰

⁸ For instance, for a satellite in a Milky Way type halo the dynamical friction time is typically of a few Gyr, while the coalescence time for black-hole binaries with masses $\sim 10^6 M_\odot$ (roughly the mass of the Milky Way MBH) is expected to be $\lesssim 10^7$ yr (Sesana, Haardt, & Madau 2007).

⁹ See however Volonteri, Haardt, & Madau (2003) for a model that does not make this assumption. That model, however, only includes dark-matter halos and MBHs, and does not attempt to describe the formation of galaxies, which makes the implementation of a realistic coalescence timescale for MBH binaries much simpler than in our case.

¹⁰ We note that while Barausse & Rezzolla (2009) also present a formula

Because during MBH mergers a fraction of the total mass is radiated in gravitational waves, the mass of the MBH remnant is smaller than the initial mass of the binary. To account for this effect, we use the formula of Reisswig et al. (2009), which accurately predicts the final mass for equal-mass black-hole binaries with spins aligned or anti-aligned with the orbital angular momentum:

$$M_{\text{bh,fin}} = [1 - (p_0 + 2p_1a + 4p_2a^2)](M_{\text{bh},1} + M_{\text{bh},2}) \quad (56)$$

where $p_0 = 4.826 \times 10^{-2}$, $p_1 = 1.559 \times 10^{-2}$, $p_2 = 0.485 \times 10^{-2}$ and $a = (a_1 + a_2)/2$ (a_1 and a_2 being the projections of the spin parameters on the direction $\hat{\mathbf{L}}$ of the orbital angular momentum). For binaries with non-aligned spins or non-equal masses, we use instead the formula of Tichy & Marronetti (2008):

$$M_{\text{bh,fin}} = (M_{\text{bh},1} + M_{\text{bh},2}) \times [1 + 4\nu(m_0 - 1) + 16m_1\nu^2(a_1 \cos \beta + a_2 \cos \gamma)] \quad (57)$$

where $m_0 = 0.9515 \pm 0.001$ and $m_1 = -0.013 \pm 0.007$.

Also, because the anisotropic emission of gravitational waves during the merger of two generic black holes carries linear momentum away from the system, the MBH remnant is imparted a recoil velocity (“kick”). There has been much controversy on the dependence of this kick velocity on the mass ratio, with Lousto & Zlochower (2009) advocating a scaling with ν^2 , and Baker et al. (2008) finding a scaling with ν^3 . It seems, however, that this discrepancy originated because Lousto & Zlochower (2009) and Baker et al. (2008) considered two different regions of the parameter space. A comprehensive formula valid in both regions was recently proposed by van Meter et al. (2010):

$$\begin{aligned} \mathbf{V}_{\text{recoil}} &= v_m \hat{\mathbf{e}}_1 + v_{\perp} (\cos \xi \hat{\mathbf{e}}_1 + \sin \xi \hat{\mathbf{e}}_2) + v_{\parallel} \hat{\mathbf{e}}_3, \quad (58) \\ v_m &= A\nu^2 \sqrt{1 - 4\nu}(1 + B\nu), \\ v_{\perp} &= H \frac{\nu^2}{1 + q} (a_1^{\parallel} - qa_2^{\parallel}), \\ v_{\parallel} &= \frac{K_2\nu^2 + K_3\nu^3}{1 + q} [qa_2^{\perp} \cos(\phi_2 - \Phi_2) - a_1^{\perp} \cos(\phi_1 - \Phi_1)] \\ &\quad + \frac{K_S(q - 1)\nu^2}{(1 + q)^3} [q^2 a_2^{\perp} \cos(\phi_2 - \Phi_2) + a_1^{\perp} \cos(\phi_1 - \Phi_1)]. \end{aligned}$$

Here $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_3$ are orthogonal unit vectors respectively in the direction of separation and along the orbital axis *just before merger*, and $\hat{\mathbf{e}}_2 = \hat{\mathbf{e}}_1 \times \hat{\mathbf{e}}_3$ is a third unit vector orthogonal to them; a_i^{\parallel} is the projection of the spin parameter \mathbf{a}_i of black hole i along the orbital angular momentum, while a_i^{\perp} is the magnitude of its projection \mathbf{a}_i^{\perp} onto the orbital plane; ϕ_i is the angle of \mathbf{a}_i^{\perp} with respect to a reference angle representing the separation of the black holes, as measured at some point before the merger, while Φ_i represents the amount by which this angle precesses before the merger.

predicting the orientation of the MBH remnant’s spin, that information is not necessary in our model. As we will mention later on, in fact, our model assumes that the spins of the two MBHs are aligned with the angular momentum of the circumbinary disk in a gas-rich environment, in which case the merger always produces a final spin in the same direction (Rezzolla et al. 2008). In a gas-poor environment, instead, we assume that the spins of the two MBHs are randomly oriented, in which case we may apply the formula of Barausse & Rezzolla (2009) for the final-spin direction. However, because accretion is chaotic in a gas-poor environment (cf. sec. 2.2.4), the information about the final spin direction is not necessary in our model.

The angles Φ_i depend on the mass ratio and on the initial separation, and must be determined with a numerical-relativity simulation (van Meter et al. 2010), which seriously undermines the predictive power of Eq. (58). The contribution v_m to the kick is dubbed the “mass-asymmetry contribution”, because it does not depend on the spins and it disappears for equal mass binaries, while v_{\perp} and v_{\parallel} are the “spin contributions”, which produce kicks perpendicular and parallel to the orbital angular momentum. The angle between the mass asymmetry and spin contributions is measured by $\xi = 215^\circ \pm 5^\circ$, while the other fitting parameters take the values $A = 1.35 \times 10^4 \text{ km s}^{-1}$, $B = -1.48$, $H = 7540 \pm 160 \text{ km s}^{-1}$, $K_2 = 3.21 \pm 0.16 \times 10^4 \text{ km s}^{-1}$, $K_3 = 1.09 \pm 0.05 \times 10^5 \text{ km s}^{-1}$ and $K_S = 1.54 \pm 0.18 \times 10^4 \text{ km s}^{-1}$.

Because it depends on quantities measured just before the black-hole merger, rather than defined at large separations (as was e.g. the case for the formula (54) for the final spin, cf. the discussion in Barausse & Rezzolla (2009)), Eq. (58) cannot be applied unambiguously in our model. In fact, the orbital axis and orbital separation directions just before merger entering Eq. (58), as well as the angles Φ_i , can in general be determined only with a full numerical-relativity simulation. An exception is given by binaries with aligned spins, in which case the magnitude of the recoil velocity is independent of ϕ_i and Φ_i , as can be seen from Eq. (58), and the orbital axis remains unchanged during the binary’s evolution. In the general case, we assume (somewhat arbitrarily, cf. Barausse & Rezzolla (2009)) that the orbital axis just before the merger is parallel to the orbital axis at large separations, and because $\phi_2 - \phi_1 = \chi$ (χ being the angle between \mathbf{a}_1^{\perp} and \mathbf{a}_2^{\perp}), defining the phases $\Delta_i \equiv \phi_i - \Phi_i$ we can write $\phi_1 - \Phi_1 = \Delta_1$ and $\phi_2 - \Phi_2 = \chi + \Delta_2$ in Eq. (58). Using (again, a bit arbitrarily, cf. Schnittman (2004); Kesden, Sperhake, & Berti (2010)) the value of χ at large separation, we only need the phases Δ_i to evaluate Eq. (58). In this paper, we choose these phases randomly from a uniform distribution.¹¹

The recoil velocity can be as large as 4000 km s^{-1} for an equal-mass binary with antialigned equal spins lying on the equatorial plane (“superkick configuration”, see Campanelli et al. (2007); González et al. (2007)). Such a large velocity would be sufficient to eject the MBH remnant from the galactic nucleus. When the spins are aligned with one another and with the orbital angular momentum, instead, the recoil velocity is much smaller and typically insufficient to allow the remnant MBH to escape from the galaxy. At each black-hole merger, we therefore check whether the MBH remnant is retained by the spheroidal component of the galaxy, thereby remaining available to accrete the cold gas brought in by radiation drag when star formation occurs in the bulge [cf. Eq. (33)], or whether it escapes. Therefore, after each black-hole merger we compare the recoil velocity V_{recoil} with the escape velocity $V_{\text{esc}} = \sqrt{2\phi}$, where ϕ is the potential due to the bulge (including both stars and gas) and the reservoir, evaluated at the galactic center. Because in the standard galaxy-formation model that we adopt here, the galactic disks form first and give rise to the bulges only later as a result of instabilities and major mergers, black-hole mergers at high redshifts will happen in disk-dominated galaxies, resulting in very large fractions of MBHs

¹¹ Another choice may be to set $\Delta_1 = \Delta_2 = 0$, and we have verified that our results do not change significantly if we make this assumption, thus confirming the intuitive expectation that the astrophysical effects of the gravitational recoil depend more on its overall scaling with the mass ratio ν than on the phases Δ_i .

	light seeds	heavy seeds
ϵ_{SN}	0.7	0.7
f_{jet}	10	10
A_{res}	1.1×10^{-2}	1.1×10^{-2}
t_{accr}	$4.8 \times 10^8 \text{ yr}$	$4.8 \times 10^8 \text{ yr}$

Table 1. The calibrated values of the free parameters of the model. These values are used to produce the figures.

being ejected. We will discuss this more in detail in Sec. 4, and hint at how this prediction may change in alternative scenarios of galaxy formation such as the “two-phase model” put forward in Cook, Lapi, & Granato (2009); Cook et al. (2010a,b). As we will see in the next sections, however, even in our current “standard” model the gravitational recoil does not eject all the MBHs from their hosts. This is because the occupation fraction of black-hole seeds at high-redshifts is smaller than 1 (cf., in Sec. 2.1, the prescriptions that we adopt to populate the halos with black-hole seeds at high redshifts), which is enough to ensure that MBHs survive to low redshifts (Lippai, Frei, & Haiman 2009; Menou, Haiman, & Narayanan 2001; Volonteri 2007; Volonteri, Gültekin, & Dotti 2010). Moreover, as emphasized by Schnittman (2007), even if we populated all halos with black-hole seeds at high redshift, the gravitational recoil in the first generation of mergers would automatically decrease the MBH occupation fraction, and even in the case of very high ejection probabilities the occupation fraction would settle to $\sim 50\%$ in a few more merger generations.

Also, we stress that our recipe to determine whether a MBH is expelled from the galaxy may be overly pessimistic. In fact, as mentioned above, here we are interested in checking whether the MBH is retained in the spheroid, because it seems reasonable that a MBH wandering in the halo would neither accrete gas from the circumnuclear reservoir, nor form MBH binaries after galaxy mergers. While this viewpoint may be sufficient for our purposes (since we want to study the mass and spin evolution of the nuclear MBHs and not of these halo MBH population), our ejection rates are clearly too large (especially at high z , when bulges are small) if one is interested in the number of MBHs that are expelled from the whole composite system (dark-matter halo and baryonic structures).

In order to apply the formulas for the MBH remnant’s mass, spin and kick velocity that we have just reviewed, we need to specify not only the masses and spins of the two progenitor MBHs, but also their relative orientation at large separations. It is well known (Bardeen & Petterson 1975; Bogdanović, Reynolds, & Miller 2007; Perego et al. 2009; Dotti et al. 2010a; Maio et al. 2012) that if the binary’s inspiral preceding the coalescence happens in a circumbinary disk (which is expected to be present in gas-rich environments), the gravito-magnetic torque exerted by the disk aligns the MBH spins with the disk’s orbital angular momentum. The details of this alignment are weakly dependent on the equation of state of the circumbinary disk, with a “cold” disk (i.e. one with polytropic index $\Gamma = 7/5$, approximating a gas with solar metallicity heated by a starburst) resulting in a residual angle $\lesssim 10^\circ$ between the spins and the disk’s angular momentum, and with a warmer disk (polytropic index $\Gamma = 5/3$, corresponding to an adiabatic monatomic gas and thus to a scenario where radiative cooling is suppressed during the merger) leading to residual angles $\lesssim 30^\circ$ (Dotti et al. 2010a). As a result, when the dynamical interaction between the binary and the gas creates a low-density region at a separation of about 0.1 pc (the so called “gap”, see Gould & Rix (2000)), the two spins are

aligned with one another and with the orbital angular momentum of the binary to within $10 - 30^\circ$. Because the gas density in the gap is very low, the subsequent evolution is driven by purely gravitational effects which tend to further align the spins (Schnittman 2004; Kesden, Sperhake, & Berti 2010). If the MBH merger happens instead in a gas-poor environment, the two spins are expected to be randomly distributed.

Existing models for the spin evolution of MBHs do not attempt to distinguish between gas-rich (“wet”) and gas-poor (“dry”) mergers. For instance, Berti & Volonteri (2008) have to consider the two possibilities separately (i.e., either all mergers are assumed to be wet or all mergers are assumed to be dry) because their model only includes dark-matter halos and MBHs and does not describe the baryonic components; Fanidakis et al. (2010, 2011) model in great detail the gravito-magnetic interaction of a single MBH with its own accretion disk, but do not include the disk’s effect on the spin alignment prior to a black-hole merger, and consider only dry mergers. Because it keeps consistently track of the evolution of the baryonic matter, our model offers a natural way to distinguish between wet and dry MBH mergers. This is important for the spin evolution of MBHs, because the spin distribution of the MBH population coming from a model with only wet mergers differs drastically from that coming from a model with only dry mergers (see Fig. 4 in Berti & Volonteri (2008)). Also, dry mergers tend to give large kick velocities, in principle sufficient to eject the remnant MBH from the galactic nucleus, because the spins prior to merger are randomly oriented. Wet mergers, where the spins are aligned, give much smaller recoil velocities (Dotti et al. 2010a). Therefore, distinguishing between wet and dry mergers is vital to predict the MBH occupation number and the number of event rates for gravitational-wave detectors.

More specifically, in our model we can discriminate between dry and wet mergers by comparing the MBH binary’s mass to that of the circumbinary disk. This comparison needs to be performed carefully, however, because as explained above the MBH merger happens *after* the galactic merger, although for simplicity this delay is not implemented in our model. Major galaxy mergers are typically accompanied by starbursts, which in our model occur because the galactic disks are disrupted and feed the gaseous bulges, where star formation is very efficient. As a result, large quantities of cold gas are forced into the circumbinary disk (i.e. the “reservoir” described in the previous sections) by radiation drag [cf. Eq. (33)]. The circumbinary disk is therefore more massive during the MBH inspiral than at the time of the galactic merger. We can therefore approximate its mass as $M_{\text{res}} + A_{\text{res}}(M_{b,\text{gas}}/t_{\text{sf}})t_{\text{ff}}$, where M_{res} and $M_{b,\text{gas}}$ are the masses of the reservoir and bulge right after the galactic merger [cf. Eqs. (52) and (53)], $M_{b,\text{gas}}/t_{\text{sf}}$ is approximately the SFR in the bulge (t_{sf} being the dynamical time of the gaseous bulge) and t_{ff} is the characteristic time of the MBH inspiral (i.e. the free fall time of the bulge, including both stars and gas). During the time t_{ff} , the two MBHs continue accreting at the expense of the circumbinary disk, and thus at a time t_{ff} after the galactic merger the mass of the binary is roughly

$$M_{\text{binary}} \approx M_{\text{bh},1} \exp\left(\frac{t_{\text{ff}}}{t_{\text{Edd},1}}\right) + M_{\text{bh},2} \exp\left(\frac{t_{\text{ff}}}{t_{\text{Edd},2}}\right) \quad (59)$$

($t_{\text{Edd},1}$ and $t_{\text{Edd},2}$ being the Eddington-accretion timescales of the

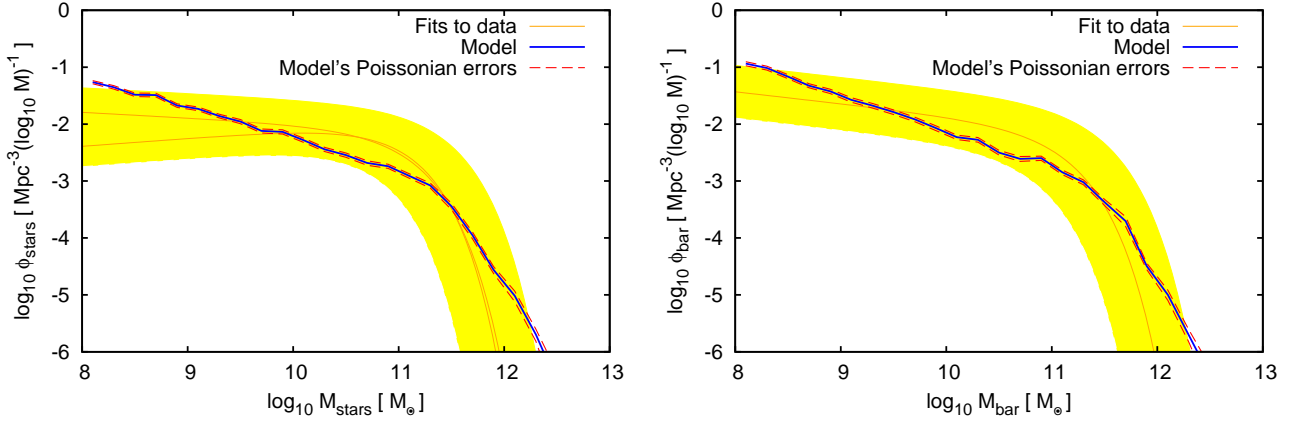


Figure 1. The local stellar mass function (left) and baryon mass function (right) of our model compared to the observational fits of Bell et al. (2003a) and Bell et al. (2003b). The observational uncertainties are shown with a shaded yellow area. These results are for the light-seed scenario, but the heavy-seed one gives similar results.

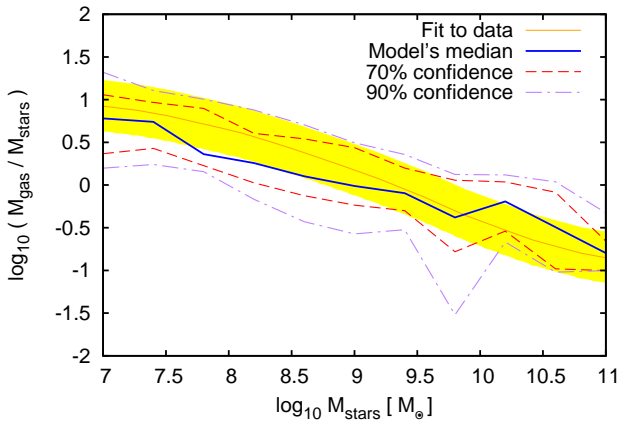


Figure 2. The local gas fraction of our model compared with the observational parameterization of Baldry, Glazebrook, & Driver (2008). The observational uncertainties are shown with a shaded yellow area. These results are for the light-seed scenario, but the heavy-seed one gives similar results.

two MBHs), to be compared with a mass

$$M_{\text{circ disk}} \approx M_{\text{res}} + A_{\text{res}} \left(\frac{M_{b,\text{gas}}}{t_{\text{sf}}} \right) t_{\text{ff}} - M_{\text{bh},1} \left[\exp \left(\frac{t_{\text{ff}}}{t_{\text{Edd},1}} \right) - 1 \right] - M_{\text{bh},2} \left[\exp \left(\frac{t_{\text{ff}}}{t_{\text{Edd},2}} \right) - 1 \right] \quad (60)$$

for the circumbinary disk. We therefore dub a merger “dry” if $M_{\text{binary}} > M_{\text{circ disk}}$, and in this case we assume that the spins of the two MBHs are randomly oriented at large separations. We instead consider a merger as “wet” if $M_{\text{binary}} < M_{\text{circ disk}}$, and in this case we assume that the spins are exactly aligned with the orbital angular momentum.

3 TESTING THE MODEL AGAINST OBSERVATIONS

Semianalytical galaxy-formation models require in general several free parameters to reproduce accurately a large range of observa-

tional data (cf. for instance Benson & Bower (2010)), which makes their calibration rather involved. Our focus here, however, is not on the galactic properties *per se*, but rather on the spin evolution of the MBH population. We therefore allow a limited number of free parameters in the galaxy formation model that we described in the previous sections, and calibrate them against a limited but representative number of observations, both at $z = 0$ and at higher redshifts.

In particular, we allow the following four free parameters to vary: (i) the supernova feedback efficiency ϵ_{SN} [cf. Eqs. (27) and (30)], which ranges from 0 to 1 and which we assume to be the same for the feedback in the disks and in the bulges; (ii) the “fudge” factor f_{jet} [cf. Eqs. (41) and (47)] parameterizing the uncertainties in the Blandford-Znajek effect (i.e. the strength of the magnetic field and the higher-order terms in the black-hole spin); this factor is assumed to vary between 0.1 and 10, and we assume it to be the same for the radio and quasar-mode feedback; (iii) the normalization factor A_{res} that describes the strength of the radiation drag regulating the growth of the circumnuclear reservoir [cf. Eq. (33)]; in order to reproduce the $M_{\text{bh}} - \sigma$ relation and the “Magorrian” $M_{\text{bh}} - M_b$ relation, it should be $A_{\text{res}} \gtrsim M_{\text{bh}}/M_b$, and because $M_{\text{bh}}/M_b \sim 10^{-3}$ (Magorrian et al. 1998; Häring & Rix 2004), we expect this parameter to be on the order of 10^{-2} – 10^{-3} (cf. also Granato et al. (2004); Lapi et al. (2006); Cook et al. (2010a,b); Haiman, Ciotti, & Ostriker (2004)); (iv) the timescale t_{accr} regulating the inflow of the circumnuclear reservoir gas into the MBH pc-scale accretion disk [cf. Eq. (34)]; because the quasar bolometric luminosity peaks at $z \approx 2$ (see e.g. Hopkins, Richards, & Hernquist (2007)), this timescale is expected to be on the order of the age of the universe at $z \approx 2$, i.e. $t_{\text{accr}} \sim 10^9$ yr. As we will show in the next sections, these four parameters are non-degenerate, because each of them (roughly) regulates the predictions for different observables, and this makes their calibration rather straightforward.

The observables that we will consider are, at $z = 0$: the stellar and baryon mass functions, the MBH mass function, the gas to stellar mass ratio, the SFR, the $M_{\text{bh}} - \sigma$ relation, the “Magorrian” $M_{\text{bh}} - M_b$ relation, and the fraction of elliptical, spiral and irregular galaxies. Also, we will test our model at $z > 0$ by looking at the SFR density (SFRD) and at the quasar bolometric luminosity density as a function of z . The four free parameters of the model

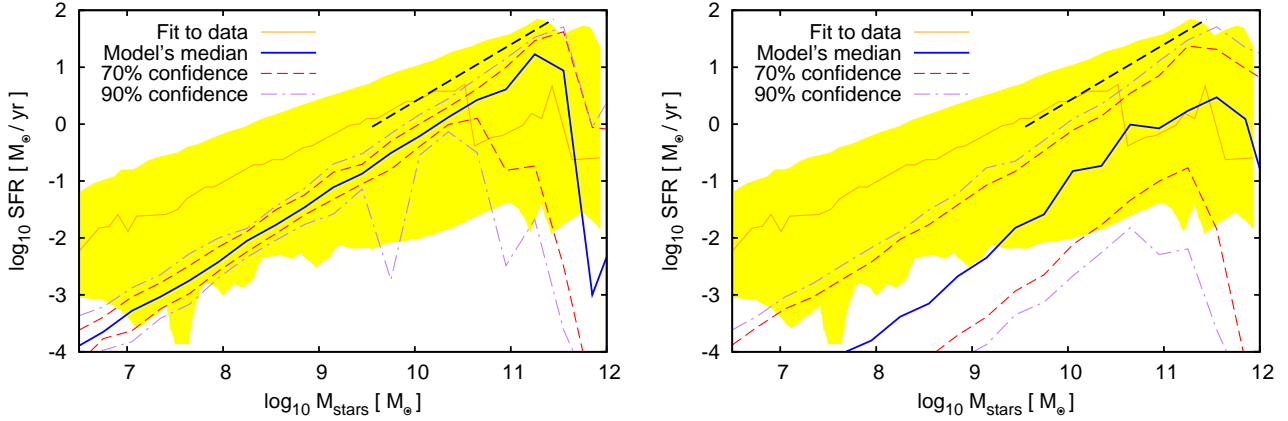


Figure 3. Our model’s SFR (at $z = 0$) compared to the observational results of Brinchmann et al. (2004). The observational 95% confidence region is shown with a shaded yellow area. In order to highlight possible selection effects, we show the output of our model including only the central galaxies with $\text{SFR} > 10^{-6} M_{\odot}/\text{yr}$ (left panel) and both the central and satellite galaxies with $\text{SFR} > 10^{-6} M_{\odot}/\text{yr}$ (right panel). These results are for the light-seed scenario, but the heavy-seed one gives similar results.

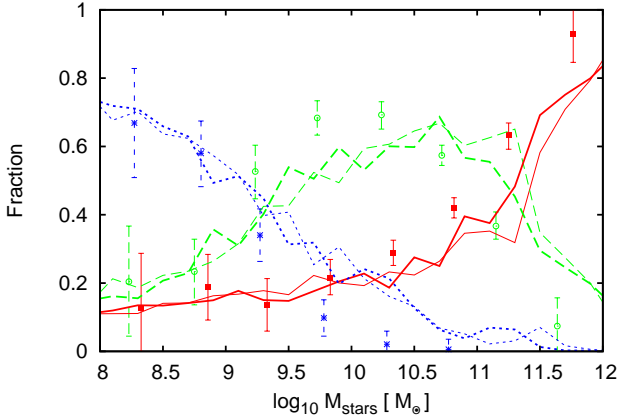


Figure 4. The fraction of elliptical, spiral and irregular galaxies, at $z = 0$, as a function of stellar mass. The symbols are data from Conselice (2006) (squares: ellipticals, circles: spirals, stars: irregulars), while the predictions of our model are shown with thick lines (heavy seeds) and thin lines (light seeds).

that we mentioned above are chosen to optimize the agreement with these observables, and their values are given in Table 1.

To obtain the results shown in the next sections, for each of our two models (light or heavy black-hole seeds) we have produced ~ 1100 halos with masses at $z = 0$ between 10^{10} and $10^{15} M_{\odot}$, and we have weighed the contribution of each one of them using the Sheth-Tormen halo mass function (Sheth & Tormen 1999, 2002) at $z = 0$. We stress that each of these halos corresponds, at $z = 0$, either to a single galaxy (if there are no satellites) or to a group or cluster of galaxies (if satellites are present).

3.1 Observables at $z = 0$

For the local stellar and baryon mass functions, we compare our results with the Schechter function fits by Bell et al. (2003a) and Bell et al. (2003b), which were obtained with a large sample of galaxies from the *Two Micron All Sky Survey* (2MASS) and the *Sloan Digital Sky Survey* (SDSS), using simple models to convert

the optical and near infrared galaxy luminosities into stellar masses and assuming a universally-applicable stellar IMF. These fits are shown in Fig. 1 (left: stellar mass function; right: baryon mass function) with orange thin solid lines, together with shaded yellow areas representing the observational errors. In particular, we have assumed 0.3 dex errors in the stellar and baryon mass determinations, and 0.3 dex in the value of the mass functions to account for the statistical errors as well as the systematic uncertainties (mainly due to the mass-to-light ratios: see discussions in Bell et al. (2003a), Bell et al. (2003b) and also Kannappan & Gawiser (2007)). Also presented in Fig. 1 is the output of our model in the light-seed scenario (the heavy-seed scenario yields similar results), together with the Poissonian errors due to the finite sample of galaxies that we simulate. As can be seen, the model’s results are generally within the observational errors, although the slope is significantly different from the observations both at the low and high-mass ends. We stress that the agreement of the model’s mass functions with the observational estimates is the product of different and competing processes. More specifically, the ionizing radiation background and the supernova feedback are effective at quenching star formation and reducing the baryonic content in small-mass systems, while the quasar and radio-mode feedback, as well as the ram pressure and clumpy accretion, are responsible for the sharp decrease of the mass function at large masses, which is significantly faster than what would be expected from the behavior of the Sheth-Tormen halo mass function alone. In particular, to reproduce the high-mass end of the stellar and baryon mass functions it is crucial to calibrate the parameter f_{jet} , which regulates the radio and quasar mode feedback in our model.

In Fig. 2 we test our model against observational results for the ratio between gaseous and stellar masses (“gas fraction”) at $z = 0$. We show the comparison for the light-seed scenario, but the heavy-seed scenario yields similar results. In particular, we use the gas-fraction parameterization of Baldry, Glazebrook, & Driver (2008) (orange thin solid line), and we represent its observational uncertainties (see Fig. 11 of Baldry, Glazebrook, & Driver (2008)) with a shaded yellow area. Because the data used by Baldry, Glazebrook, & Driver (2008) include only field galaxies, when calculating the gas fraction in our model we only include central galaxies whose stellar mass is at least 90% of the total stellar

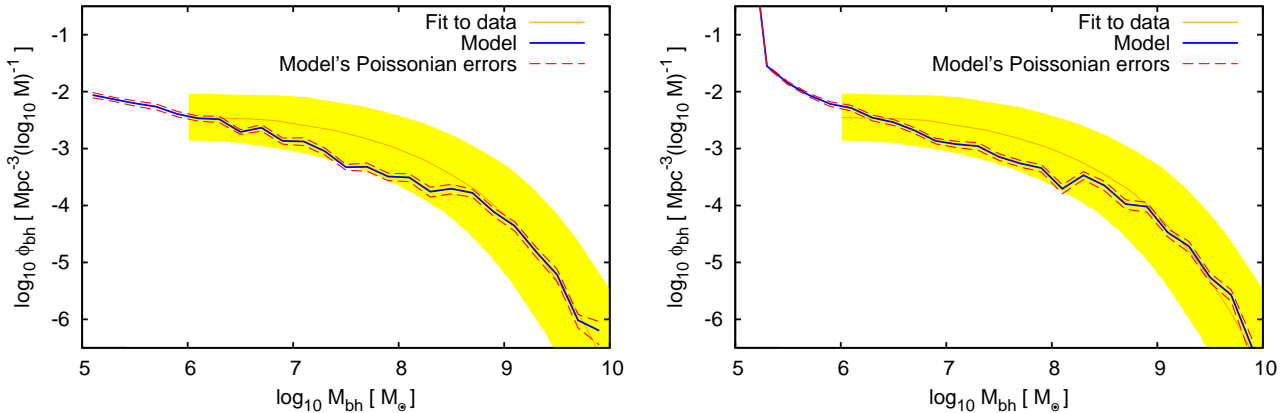


Figure 5. The local MBH mass function predicted by our model in the light-seed (left) and heavy-seed (right) scenario, compared to the observational estimate by Marconi et al. (2004). The observational uncertainties are shown with a shaded yellow area. These results are for the light-seed scenario, but the heavy-seed one gives similar results.

mass of the system (central galaxy plus all the satellites). Also, the data used by Baldry, Glazebrook, & Driver (2008) do not include gas-poor ellipticals, but those galaxies are only a significant field population at $M_{\text{stars}} \gtrsim 10^{11} M_{\odot}$, which is outside the range of Fig. 2. Because of the scatter of our model’s predictions, we present its median output at a given stellar mass, as well as its 70% and 90% confidence-level regions, i.e. the regions containing respectively 70% and 90% of the galaxies produced by our model at a given stellar mass. As can be seen, the model is in good agreement with the observational data. In particular, in order to obtain a good agreement with the observed gas fraction at $M_{\text{stars}} \sim 10^7 - 10^8 M_{\odot}$, it is crucial to tune the parameter ϵ_{SN} , which regulates the efficiency of the supernova feedback.

The SFR at $z = 0$ as a function of stellar mass, as obtained from a survey $\sim 10^5$ galaxies with measurable SFR in the SDSS (Brinchmann et al. 2004), is shown in Fig. 3. The orange thin solid line represents the median SFR observed at a given stellar mass, while the shaded yellow area represents the observational 95% confidence level (i.e. the region containing 95% of the galaxies at a given stellar mass.). The median SFR predicted by our model at a given stellar mass (in the light-seed scenario: the heavy-seed one gives again similar results) is represented by the thick solid blue line, while the thin red dashed and thin purple dot-dashed lines represent the model’s 70% and 90% confidence regions. In particular, because the median and confidence region of Brinchmann et al. (2004) may depend on the composition of their sample of galaxies, we have plotted the output of our model when considering only the central galaxies, where star formation is more intense (left panel), and when considering both the central galaxies and the satellites (right panel). In both cases, we have neglected galaxies with SFR less than $10^{-6} M_{\odot}/\text{yr}$, since the sample of Brinchmann et al. (2004) only includes galaxies with measurable SFR.

As can be seen, irrespective of which of the two samples we consider, the median predictions of our model lie within the 95%-confidence region of the observational data, and reproduce the flattening of the SFR at high masses (due to the combined effect of quasar and radio-mode feedback, clumpy accretion and ram pressure). However, our predictions are significantly lower than the median of the observations, especially at small stellar masses, essentially because they present a steeper slope. More specifically, our

model predicts $\text{SFR} \propto M_{\text{stars}}$, while the data by Brinchmann et al. (2004) suggest $\text{SFR} \propto M_{\text{stars}}^n$, with $n \approx 0.7$. While other observational data point at a slope n slightly less than 1 (for instance, Elbaz et al. (2007) find $n \approx 0.77$ and Salim et al. (2007) find $n \approx 0.65$), Elbaz et al. (2011) find that at $z = 0$ the $\text{SFR}-M_{\text{stars}}$ relation is well fitted by $\text{SFR} = M_{\text{stars}} / (4 \times 10^9 \text{ yr})$ for $9.5 \lesssim \log_{10} M_{\text{stars}} \lesssim 11.5$. This parameterization is represented in Fig. 3 with a dashed thick black line. Also, there is some evidence that the slope n is rather sensitive to selection effects: for instance, Karim et al. (2011) find that for highly active star-forming galaxies the exponent n approaches 1, although it is not clear whether these galaxies are representative of the entire star-forming population. In general, the slope $n \approx 1$ of our model’s predictions is due to our star formation prescriptions of Sec. 2.2.3, and is therefore common to most semianalytical galaxy-formation models (see for instance Dutton, van den Bosch, & Dekel (2010)). While it is in principle possible to change our star formation prescription to obtain a milder slope (Shi et al. 2011), at this point it is not clear whether this is needed, because it seems that the issue is still not settled from the observational point of view.

In Fig. 4, we plot the fraction of elliptical, spiral and irregular galaxies as a function of stellar mass. The symbols are actual data from Conselice (2006) (squares: ellipticals, circles: spirals, stars: irregulars), while the predictions of our model are shown with thick lines (heavy seeds) and thin lines (light seeds). More specifically, within our model we follow Guo et al. (2011) and use the ratio M_b/M_{tot} (with $M_{\text{tot}} = M_b + M_d$) to discriminate different morphologies. In particular, we identify galaxies with $M_b/M_{\text{tot}} > 0.7$ with ellipticals and represent them with solid lines; galaxies with $0.03 < M_b/M_{\text{tot}} < 0.7$ with spirals and represent them with dashed lines; galaxies with $M_b/M_{\text{tot}} < 0.03$ with extreme late-type or pure-disk galaxies (and therefore with irregulars (Guo et al. 2011)), and represent them with dotted lines. Remarkably, in spite of this simplistic classification, our model seems to reproduce the observed morphological fractions, at least qualitatively.

To test our predictions for the MBH population at $z = 0$, we look at the local MBH mass function, at the $M_{\text{bh}} - \sigma$ relation between the black-hole mass and the line-of-sight velocity dispersion σ of the bulge (Ferrarese & Merritt 2000; Gebhardt et al. 2000; Gültekin et al. 2009; Graham 2008; Graham et al. 2011; Hu 2008), and at the relation between the MBH mass and the bulge

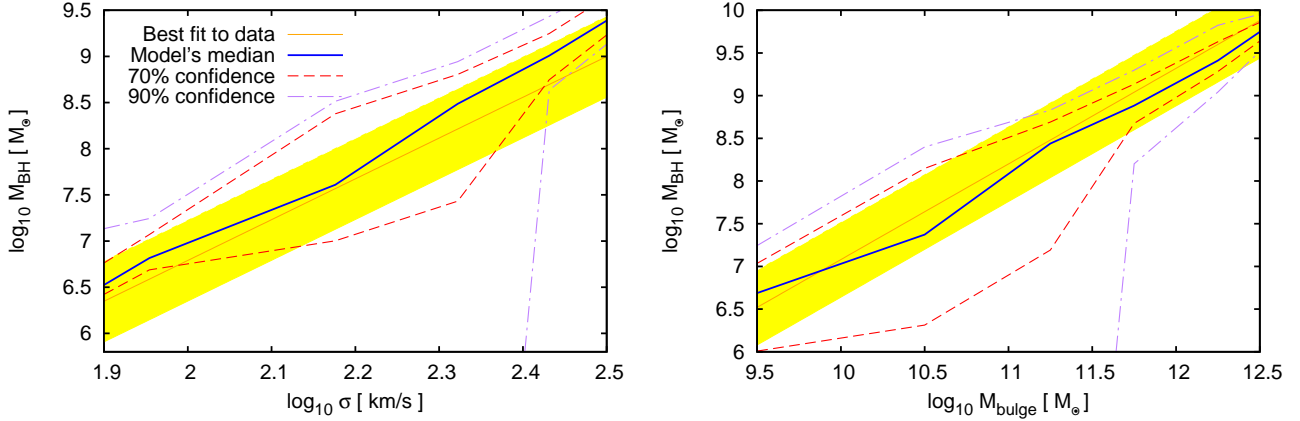


Figure 6. The predictions of our model, in the light-seed scenario, for the $M_{\text{bh}} - \sigma$ and $M_{\text{bh}} - M_b$ relations at $z = 0$, compared to the observational fits of Gültekin et al. (2009) and Häring & Rix (2004), respectively. The observational uncertainties are shown with a shaded yellow area.

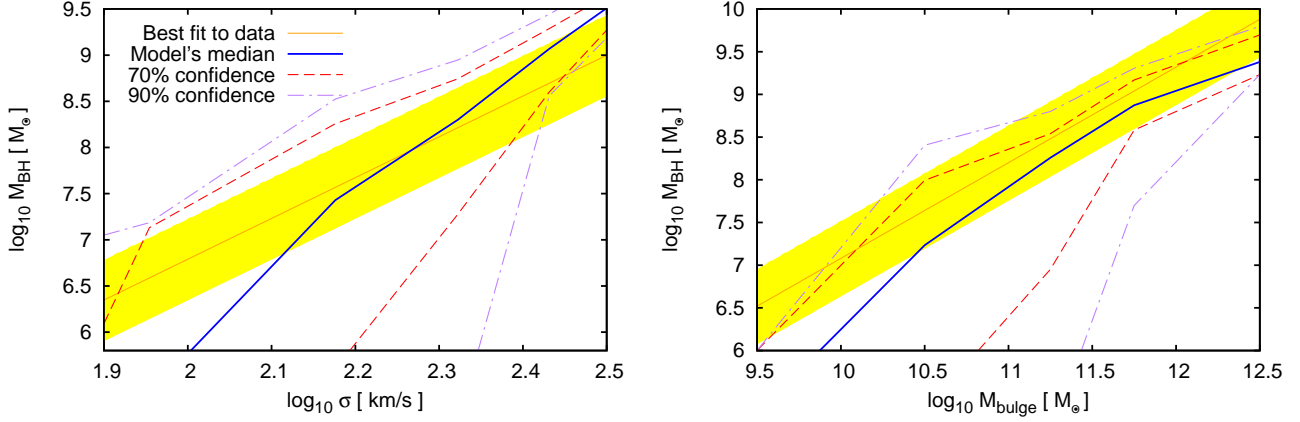


Figure 7. The same as in Fig. 6, but for the heavy-seed scenario.

mass, initially proposed by Magorrian et al. (1998) and later updated by Häring & Rix (2004). The predictions of our model for these observables mainly depend on the normalization factor A_{res} regulating the growth of the circumnuclear reservoir and therefore of the MBHs, and we calibrate this parameter to obtain agreement with the observations. In particular, in Fig. 5 we show the estimate by Marconi et al. (2004) for the MBH mass function at $z = 0$ with a thin solid orange line, and we assume observational uncertainties, represented by shaded yellow areas, of 0.3 dex for the MBH mass and 0.4 dex for the mass function (cf. Shankar, Weinberg, & Miralda-Escudé (2009), Fig. 5). The predictions of our model are instead shown in the left panel for the light-seed scenario, and in the right-panel for the heavy-seed scenario, with error bars representing the Poissonian errors due to finite sample of galaxies that we simulate. As can be seen, both scenarios produce MBH mass functions that are compatible with the observational estimate, but they yield different predictions for $M_{\text{bh}} \lesssim 10^6 M_{\odot}$, where the mass function is unconstrained by observations. In particular, the heavy-seed scenario obviously predicts a large number of MBH with $M_{\text{bh}} \gtrsim M_{\text{seed}} = 10^5 M_{\odot}$, while the light-seed scenario predicts a flatter mass function.

In the left panel of Figs. 6 (light-seed scenario) and 7 (heavy-seed scenario), we show the parameterization of Gültekin et al.

(2009) for the $M_{\text{bh}} - \sigma$ relation originally discovered by Ferrarese & Merritt (2000); Gebhardt et al. (2000), while in the right panel we show the parameterization of Häring & Rix (2004) for the $M_{\text{bh}} - M_b$ relation, originally suggested by Magorrian et al. (1998). In particular, both parameterizations are represented by thin solid orange lines, while the 1σ observational scatter is denoted by a yellow shaded area. We also show the median of our model's predictions for M_{bh} at any given σ (M_b), as well as their 70% and 90% confidence bands.

More specifically, in order to calculate the median and confidence regions in our model we have neglected the MBHs residing in satellite galaxies. Also, even for the central galaxies, we have only considered the MBHs residing in elliptical galaxies (which we identify again with ones having $M_b/M_{\text{tot}} > 0.7$ (Guo et al. 2011)). This is because the $M_{\text{bh}} - \sigma$ relation is known to present a large scatter for late-type galaxies (cf. discussion in Gültekin et al. (2009) and their Fig. 1; see also Graham (2008); Graham et al. (2011); Hu (2008)), and the $M_{\text{bh}} - M_b$ relation has been established in Häring & Rix (2004) using a sample of 30 mostly elliptical galaxies. The predictions of our model are obtained using the virial theorem to relate the bulge velocity dispersion V_{bulge}^2 to the bulge gravitational potential (which we calculate from the bulge density (21)), and then calculating the line-of-sight velocity disper-

sion as $\sigma^2 = V_{\text{bulge}}^2/K$, where the correction factor K depends on the type of orbits of the stars in the bulge. For instance, if the orbits were mainly along the line of sight, one would have $K \approx 1$, but $K \approx 3$ for nearly circular and isotropic orbits. Here, following Rood (1970), we assume $K = 2.1$. Overall, as can be seen, both the light-seed and heavy-seed scenario reproduce the observational relations only qualitatively, because they predict a significant number of “outliers” below the observed correlations, with masses $M_{\text{bh}} \lesssim 10^8$. These outliers become even more numerous if one includes the spiral and irregular galaxies in the analysis, or if one also accounts for the MBHs in satellite galaxies. Physically, they therefore represent MBHs that will settle on the $M_{\text{bh}} - \sigma$ and $M_{\text{bh}} - M_b$ relations in the future, after merging with a MBH residing in a central galaxy already on the $M_{\text{bh}} - \sigma$ and $M_{\text{bh}} - M_b$ relations, or after a burst of star formation in the bulge following a disk disruption or a major merger. This population of outliers is also found for instance in the model of Volonteri & Natarajan (2009), but remarkably not in the “two-phase” galaxy-formation model proposed by Cook, Lapi, & Granato (2009); Cook et al. (2010a,b) (see in particular Fig. 7 of Cook et al. (2010b)), where bulges form at high redshifts and MBHs settle on the $M_{\text{bh}} - \sigma$ and $M_{\text{bh}} - M_b$ relations earlier. We will discuss this “two-phase” model and its possible consequences on the MBH redshift evolution more in detail in Sec. 4.

3.2 Observables at $z > 0$

In this section we will examine the output of our model for two observables at $z > 0$, namely: the SFR density (SFRD) and the bolometric quasar luminosity density.

There are at least two classes of methods to determine the redshift evolution of the SFRD, also known as the star-formation “cosmic history” (Madau et al. 1996; Lilly et al. 1996). On the one hand, the SFRD can be measured directly at various redshifts using instantaneous indicators. More specifically, emission from massive stars, which are short-lived compared to the typical star-formation timescales, can be used to extrapolate the IMF and obtain the SFR (see Hopkins (2004, 2007); Hopkins & Beacom (2006, 2008) for compilations of recent data). A similar procedure can be followed with core-collapse supernova rates (see for instance Dahlen et al. (2004)), for these stars too are short-lived compared to star formation. Alternatively, one can measure the distribution of stellar ages in nearby galaxies and use stellar population synthesis models to reconstruct the star-formation cosmic history (see Panter et al. (2007) for a recent analysis).

On the other hand, integrating the cosmic star-formation history over redshift and correcting for the mass loss through supernovae and stellar winds yields the stellar-mass density as a function of redshift. Conversely, the cosmic star-formation history can be extracted from the stellar-mass density evolution, which can be measured independently with galaxy surveys and is sensitive to a larger range of masses than instantaneous indicators, which only probe the most massive stars. Moreover, instantaneous indicators are subject to greater uncertainties due to the effects of dust obscuration. A compilation of recent measurements of the stellar-mass density as a function of redshift is presented by Wilkins, Trentham, & Hopkins (2008), who showed that the inferred star-formation history agrees with the one derived from instantaneous indicators for $z < 0.7$, at least for suitable choices of the IMF (see also Hopkins & Beacom (2008)). At higher redshift, however, the instantaneous indicators give larger SFRDs than the evolution of the stellar-mass density,

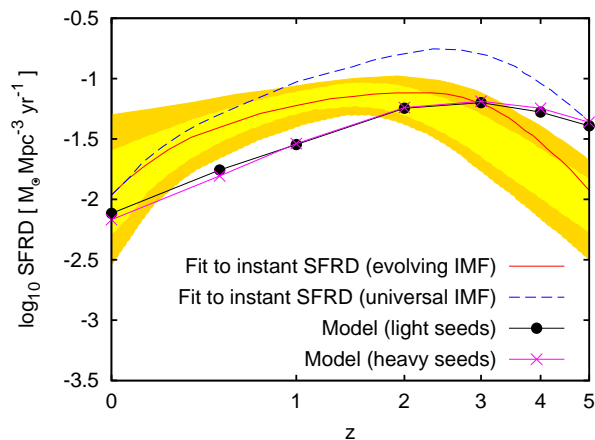


Figure 8. The cosmic star-formation history predicted by our model, compared to that derived by Wilkins, Trentham, & Hopkins (2008) from observations of the stellar mass-density and assuming an evolving IMF (yellow and orange shaded areas, representing the 1σ and 3σ uncertainty regions of the observations). Also shown are fits to instantaneous SFRD indicators (Wilkins, Trentham, & Hopkins 2008), assuming either a universal or an evolving IMF.

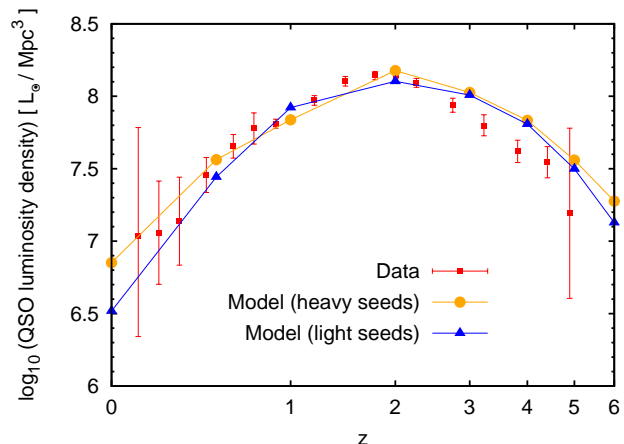


Figure 9. The prediction of our model for the bolometric quasar luminosity density as a function of redshift, compared to data from Hopkins, Richards, & Hernquist (2007).

and this discrepancy peaks at $z \approx 3$, where the difference is ~ 0.6 dex (Wilkins, Trentham, & Hopkins 2008; Wilkins et al. 2008).

There are various possible explanations for this discrepancy, namely uncertainties in the effect of dust on the stellar-mass and SFR estimates at high-redshifts (Driver et al. 2007), or incompleteness in the measured stellar-mass density (Reddy & Steidel 2009). Another possibility is that the IMF might evolve with redshift (Wilkins, Trentham, & Hopkins 2008; Wilkins et al. 2008; Davé 2008), and there are in fact theoretical arguments and indirect observational evidence for that (van Dokkum 2008; Larson 2005; Wang & Dai 2011). In Fig. 8, the yellow and orange shaded areas represent the 1σ and 3σ uncertainty regions of the cosmic star-formation history derived by Wilkins, Trentham, & Hopkins (2008) from the evolution of the stellar mass-density, assuming an evolving IMF. The dashed and solid lines are Wilkins, Trentham, & Hopkins (2008)’s fits to instantaneous SFRD indicators, assuming respectively a universal and an evolving IMF. In this figure, we also show the output of our model. As can be

seen the light-seed and heavy-seed scenarios predict essentially the same SFRD, but these predictions agree only qualitatively with the observations. While it should be possible to amend our model to reproduce the observational data more closely, that would probably require including more free parameters than the four that we consider in this paper (as of now, we have no parameters regulating the cosmic star-formation history directly, besides those that we fixed by comparing to the $z = 0$ observations of the previous section). We deem such a refinement premature due the discrepancies between different indicators and the problems in interpreting the SFRD observations that we mentioned above.

Finally, in Fig. 9 we show data from Hopkins, Richards, & Hernquist (2007) for the bolometric quasar luminosity density as a function of redshift, and the corresponding output of our model. As can be seen both the light-seed and heavy-seed scenarios do a good job at reproducing the observations, and in particular the peak of the quasar luminosity at $z \approx 2$. We stress that while the normalization of the quasar luminosity density depends on the parameter A_{res} , which also regulates the normalization of the MBH mass function and that of the $M_{\text{bh}} - \sigma$ and $M_{\text{bh}} - M_b$ relations (see previous section), its shape crucially depends on our free parameter t_{accr} , which we calibrate to reproduce the observations. Also, the predictions of our model as shown in Fig. 9 only consider MBHs with bolometric luminosities $L > 10^{10} L_{\odot}$. This is because lower luminosities cannot be observationally resolved as confusion with normal star-forming and starburst galaxies becomes an issue, and therefore they do not enter in the analysis of Hopkins, Richards, & Hernquist (2007).

4 THE CHARACTER OF MBH MERGERS

In this section we examine the predictions of our model for the MBH mergers, focusing in particular on their character, i.e. whether they happen in gas-rich (“wet”) nuclear environments (where the gravito-magnetic torques align the MBH spins prior to the merger) or in gas-poor (“dry”) ones, where the spins prior to the merger are randomly oriented.

In Figs. 10 (light-seed scenario) and 11 (heavy-seed scenario) we present predictions for the number of mergers observed in 1 yr at $z = 0$, for the fraction of MBH remnants that are ejected from galactic spheroids¹² as a result of the gravitational recoil, and for the fraction of wet mergers, in different ranges of the MBH binary’s mass $M_{\text{bin}} = M_{\text{bh}1} + M_{\text{bh}2}$ and as a function of redshift. The predictions for the number of mergers, however, might be regarded as lower limits to the actual rates, because of the prescriptions described in Sec. 2.1 and aimed at keeping the computational time needed to follow the merger trees up to high redshifts to an acceptable level.

In spite of this note of caution, the results for the light-seed scenario seem to confirm that LISA/SGO or a similar European mission such as eLISA/NGO should detect at least a few MBH merger events during its lifetime, although a detailed analysis will be necessary when the details of the mission (e.g. its duration and sensitivity curve) are finalized. In fact, focusing for the moment on the $10^4 M_{\odot} < M_{\text{bin}} < 10^6 M_{\odot}$ and $M_{\text{bin}} > 10^6 M_{\odot}$ mass ranges

(which are the most relevant for LISA/SGO and eLISA/NGO), we notice that the merger rates shown in Fig. 10 are qualitatively similar to the predictions of Sesana, Volonteri, & Haardt (2007) for the “BVRhf” model (cf. their Fig. 1), which yields event rates of a few per year for the original LISA mission. In the heavy-seed scenario (Fig. 11), the events in the mass range $10^4 M_{\odot} < M_{\text{bin}} < 10^6 M_{\odot}$ are instead much more numerous (~ 200 per year for $z < 10$). This is indeed in agreement with the analysis of Sesana, Volonteri, & Haardt (2007), who found that models with heavy seeds and large initial seed occupation numbers (i.e. their “KBD” model) should give event rates of hundreds per year for LISA/SGO or eLISA/NGO. As for the low-mass range $M_{\text{bin}} < 10^4 M_{\odot}$, our light-seed scenario predicts about 25 mergers per year for $z < 10$. (Clearly, mergers in this mass range are not present in the heavy-seed scenario.) These mergers may give a significant event rate for future third generation gravitational-wave detectors in the 0.1-10 Hz frequency band, such as DECIGO or the Einstein Telescope (Sesana et al. 2009; Gair et al. 2009, 2011), or might even be marginally detectable with LISA/SGO or eLISA/NGO at high redshifts if they have $M_{\text{bin}} \lesssim 10^4 M_{\odot}$.

As for the fraction of MBHs ejected from galactic bulges and for the fraction of wet mergers, Figs. 10 and 11 confirm one’s intuitive expectations. In the light-seed scenario, the mergers in the $M_{\text{bin}} < 10^4 M_{\odot}$ mass range almost always result in the MBH remnant being ejected. This is because the mass of MBHs remains small (i.e. $\sim M_{\text{seed}}$) only if the host galaxies are disk-dominated, in which case little or no star formation happens in the bulge, and no cold gas becomes available for the MBHs to grow. Naturally, if the bulge components are small they can hardly retain the MBHs. Also, because the MBH seeds all have the same mass, mergers between seeds have mass ratio $q \sim 1$, which gives larger recoil velocities (cf. the mass-ratio dependence of Eq. (58)). Mergers in the $10^4 M_{\odot} < M_{\text{bin}} < 10^6 M_{\odot}$ and $M_{\text{bin}} > 10^6 M_{\odot}$ mass ranges, instead, are less likely to result in MBH ejections in the light-seed scenario. This is because if the MBHs have managed to grow beyond mere seeds, they have done so by accreting the cold gas brought to galactic nuclei by star formation in the bulges (via radiation drag). As a result, the bulges are more massive than in the case of mergers of MBH seeds, and they are more likely to retain the MBH remnants resulting from mergers. Also, because the MBH have grown to masses far from that of their seeds, the mergers are likely to involve mass ratios significantly different from $q = 1$. This is indeed shown in Fig. 12, where we show the different mass ratios occurring in MBH mergers in the light-seed scenario, in different mass ranges. As can be seen, for $M_{\text{bin}} < 10^4 M_{\odot}$ the coalescing black holes often have comparable masses (essentially because many of them are still seed black holes, or have grown little away from them), while the mass ratios become more varied at higher masses.

Similarly, as can be seen from Fig. 10, the mergers in the mass range $M_{\text{bin}} < 10^4 M_{\odot}$ are wet only in $\sim 30 - 40\%$ of cases in the light-seed scenario, because if a significant amount of gas were present in the galactic center, the black holes would have rapidly grown beyond $10^4 M_{\odot}$. In the $10^4 M_{\odot} < M_{\text{bin}} < 10^6 M_{\odot}$ and $M_{\text{bin}} > 10^6 M_{\odot}$ mass ranges, the mergers are mainly wet at high redshifts, where a lot of cold gas is present in galactic nuclei, but the fraction of wet mergers decreases as the cosmic evolution progresses, because the amount of gas shrinks as a result of both accretion by the MBH and its quasar and radio-mode feedback on the galaxy (as well as a result of ram pressure and clumpy accretion, at large halo masses and low redshifts). Such a decrease in the fraction of gas-rich MBH mergers with redshift is a prediction that can be in

¹² As mentioned in Sec. 2.3.1, our results for the ejection rates are overly pessimistic if one is instead interested in the fraction of MBHs that are ejected from the whole composite system (dark-matter halo and baryonic structures).

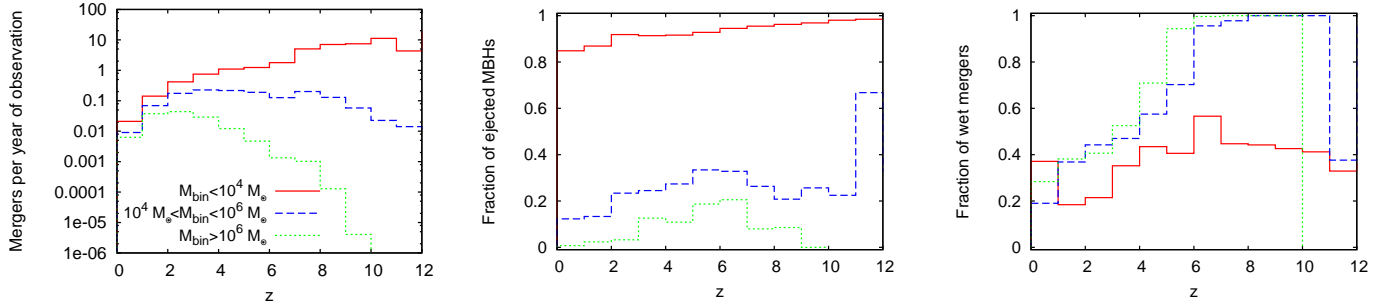


Figure 10. The predictions of our model (in the light-seed scenario) for the number of MBH mergers per year of observation, for the fraction of mergers producing a MBH that is ejected from the galaxy as a result of the gravitational recoil, and for the fraction of gas-rich (“wet”) MBH mergers, as a function of redshift and in different mass ranges. We notice that the number of mergers with $M_{\text{bin}} > 10^6 M_{\odot}$ drops to zero for $z > 10$, hence the fraction of wet mergers and that of ejected MBHs are not defined for $z > 10$ in that mass range.

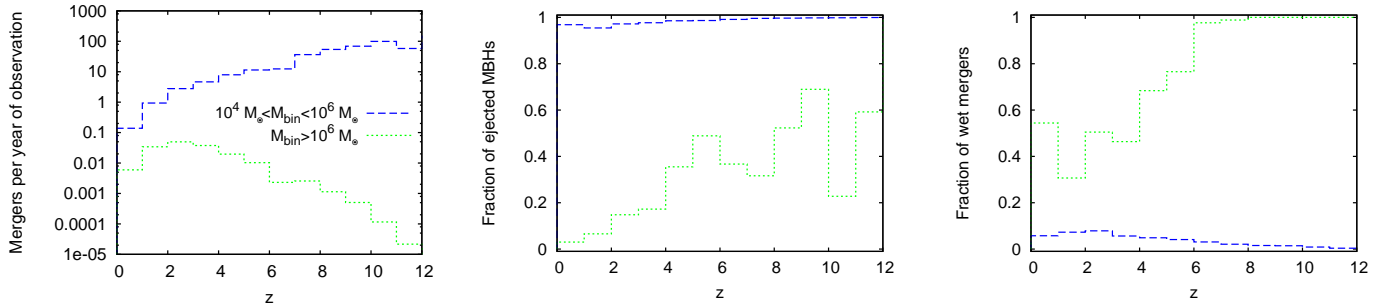


Figure 11. The same as in Fig. 10, but in the heavy-seed scenario. We notice that no mergers with $M_{\text{bin}} < 10^4 M_{\odot}$ are present, because of the seed model.

principle tested with gravitational-wave missions (e.g. LISA/SGO or eLISA/NGO). As already mentioned, the MBHs spins tend to be at least partially aligned in gas-rich environments due to the gravito-magnetic torques exerted by the gas, while the spins are expected to be randomly oriented and precess more strongly in gas-poor ones. Missions like LISA/SGO or eLISA/NGO should be able to tell a binary with nearly aligned spins from a binary with strongly precessing ones by looking at the higher-order harmonics of the gravitational waveforms (Lang & Hughes 2006, 2007, 2008; Lang, Hughes, & Cornish 2011).

The situation is similar in the heavy-seed scenario, as can be seen from Figs. 11 and 13. The mergers in the $10^4 M_{\odot} < M_{\text{bin}} < 10^6 M_{\odot}$ mass range happen between MBHs that have barely grown away from their original seeds of mass $M_{\text{seed}} = 10^5 M_{\odot}$. Therefore, these mergers usually present comparable masses, happen in dry environments, and likely eject the resulting MBH from the galactic spheroid. The mergers in the $M_{\text{bin}} > 10^6 M_{\odot}$ mass range, instead, consist of MBHs that have grown significantly larger than their seeds thanks to a gas rich environment and to significant star formation in the bulge, and therefore tend to be retained in spheroids after mergers, and to merge in “wet” environments (at least at high redshifts, where circumbinary disks have not yet been destroyed by accretion, AGN feedback, ram pressure and clumpy accretion).

We stress that in spite of the rather high ejection probability in the mergers between seeds that one can observe in Figs. 10 and 11, our model correctly reproduces the property of MBHs at low redshifts, as we showed in the previous section. In other words, the gravitational recoil does not succeed at rooting out all the MBHs from their host galaxies. This is essentially because

the occupation fraction of black-hole seeds at high-redshifts is smaller than 1 (cf., in Sec. 2.1, how we populate high-redshift halos with black-hole seeds), and this is enough to ensure that MBHs survive to low redshifts (Lippai, Frei, & Haiman 2009; Menou, Haiman, & Narayanan 2001; Volonteri 2007; Volonteri, Gültekin, & Dotti 2010). Also, as noted by Schnittman (2007), even if we populated all halos with black-hole seeds at high redshift, the gravitational recoil in the first generation of mergers would automatically decrease the MBH occupation fraction, and even in the case of very high ejection probabilities the occupation fraction would settle to $\sim 50\%$ in a few more merger generations.

Another possibility would be to adopt the galaxy-formation model of Cook, Lapi, & Granato (2009); Cook et al. (2010a,b), in which the baryonic evolution is driven by the two-phase structural evolution of the dark-matter haloes (Zhao et al. 2003a,b; Mo & Mao 2004; Diemand, Kuhlen, & Madau 2007), and presents two distinct phases: an early “fast collapse” phase, where the dark-matter core structure is built through a series of violent merger events, corresponding to an epoch where baryonic material collapses to directly form spheroidal structures (bulges); and a late “slow collapse” phase, where potentially large amounts of material are added to the halo outskirts without affecting the central regions, giving rise to the quiescent growth of disk structures around the previously-formed bulges. This model can potentially reproduce the observed downsizing of baryonic structures more naturally than standard semianalytical galaxy-formation models (Cook, Lapi, & Granato 2009), and may potentially have effects on the predictions for the MBH merger rates and mass evolution. As already mentioned in Sec. 3, the standard galaxy-formation model that we adopt here, in which disk galaxies form

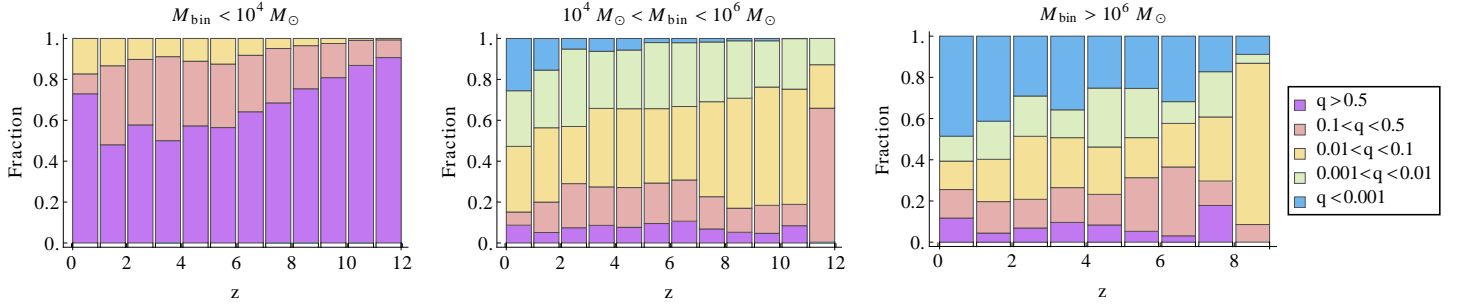


Figure 12. The predictions of our model (in the light-seed scenario) for the distribution of mass ratios $q = M_{\text{bh},2}/M_{\text{bh},1}$ (where $M_{\text{bh},2} \leq M_{\text{bh},1}$) in MBH mergers, as a function of redshift and in different mass ranges.

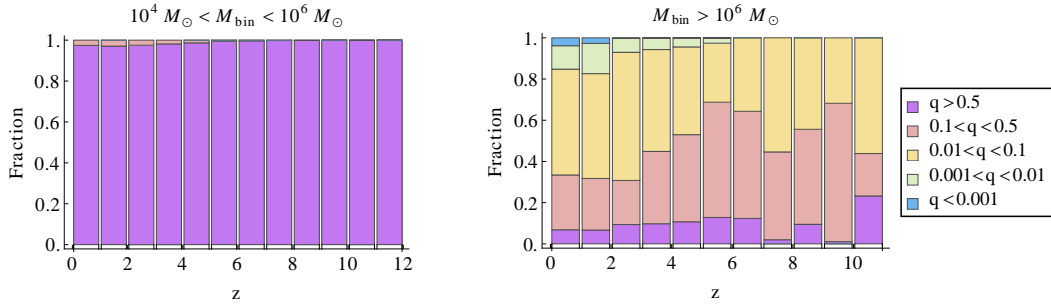


Figure 13. The same as in Fig. 12, but in the heavy-seed scenario.

first and give rise to spheroids by instabilities and major mergers, predicts the presence of a significant number of outliers in the $M_{\text{bh}} - \sigma$ and $M_{\text{bh}} - M_b$ relations, while MBHs settle on the $M_{\text{bh}} - \sigma$ and $M_{\text{bh}} - M_b$ relations earlier in the case of this “two-phase” model (see Cook et al. (2010b)). This is because spheroidal structures form first, and as a result of radiation drag they feed the MBHs at higher redshifts than in the “standard” model. Clearly, in such a scenario one does not have high-redshift mergers between galaxies with little or no bulges, and therefore the ejection rate of black-hole seeds should be significantly reduced. Also, the earlier growth of the MBHs may boost the event rates for LISA/SGO or eLISA/NGO in the light-seed scenario. We will explore in detail the effects of this alternative galaxy-formation model on the MBH mergers and evolution in a future paper.

Finally, as mentioned in the introduction, the results of this section depend on our assumption that the MBH spins get aligned by gravitomagnetic torques in gas-rich environments. However, due to processes such as star formation and feedback in the circumnuclear disk (Dotti et al. 2010a; Maio et al. 2012) or clump formation in high- z disk galaxies (Bournaud et al. 2011; Dubois et al. 2011), this alignment is likely to be only partial, which may result in higher ejection rates for the MBHs resulting from mergers.

5 THE EVOLUTION OF THE MBH SPINS

As a final application of our model, we study the redshift evolution of the MBH spins. These predictions will be readily testable by LISA/SGO or eLISA/NGO, which will measure the black-hole masses and spins with astonishing accuracy ($\sim 10^{-3}$ for the masses and 10^{-2} for the spins, see Berti, Buonanno, & Will (2005); Lang & Hughes (2006, 2007, 2008); Lang, Hughes, & Cornish (2011)) and without the system-

atic uncertainties typically affecting electromagnetic (e.g. X-ray) determinations.

In Figs. 14 (light-seed scenario) and 15 (heavy-seed scenario), we present results for the distribution of masses and spins of the MBHs residing in isolated galaxies or in the central galaxies of groups or clusters (i.e., we do not consider the MBHs residing in satellite galaxies), at redshifts ranging from $z = 7$ to $z = 0$. The color code represents the \log_{10} of the density of MBHs per unit (logarithmic) mass and unit spin, $\log_{10}(d\phi_{\text{bh}}[\text{Mpc}^{-3}]/da) = \log_{10}(d^2n_{\text{bh}}[\text{Mpc}^{-3}]/(d\log_{10} M_{\text{bh}}[M_{\odot}] da))$. As can be seen, in the light-seed scenario, already at $z = 7$ the MBH distribution has been skewed towards large spins from the initial uniform spin distribution of the seeds (still visible at $M_{\text{seed}} = 150M_{\odot}$). This is because at high redshifts, where the AGN feedback is still ineffective and the MBHs small, large amounts of gas are present in galactic nuclei, and the MBH spins grow as a result of wet, spin-aligned mergers (cf. Sec. 4) and most importantly because accretion onto the MBHs is coherent and spins them up (cf. Sec. 2.2.4). We stress, as already mentioned in the introduction, that effects such as star formation and feedback in the circumnuclear disk, or the formation of clumps in high-redshift disk galaxies fed by cold streams, may at least partially randomize the accretion flows in gas-rich environments. As a result, the large spins $a_{\text{bh}} \sim 1$ shown in Figs. 14 and 15 at high redshift may be substantially reduced (e.g. Maio et al. (2012) show that for quasars in merger remnants, sustained accretion results asymptotically in spins parameters $a_{\text{bh}} \sim 0.7 - 0.9$).

At smaller redshifts this trends gets modified because the cold gas in the nuclear regions of galaxies becomes scarcer, hence mergers tend to happen in dry environments (cf. Sec. 4) and accretion turns chaotic. Chaotic accretion, in particular, appears to be the main driving force behind the spin evolution in this phase, as can be seen from the appearance of a large number of MBHs with

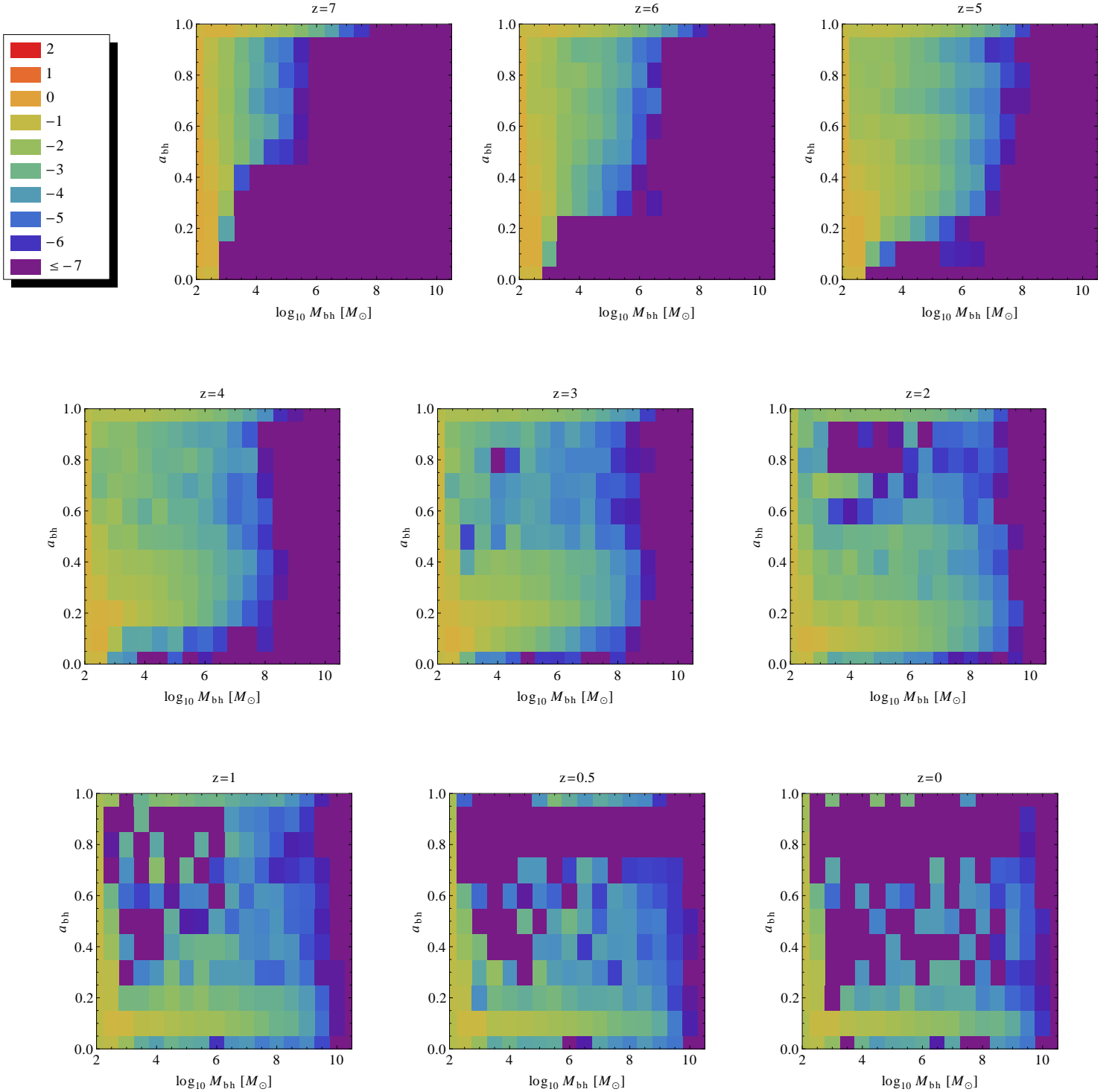


Figure 14. The evolution of the MBH masses and spins with redshift, as predicted by our model in the light-seed scenario. The color code represents the \log_{10} of the density of MBHs per unit (logarithmic) mass and unit spin, i.e. $\log_{10}(d\phi_{bh}[\text{Mpc}^{-3}]/da) = \log_{10}(d^2n_{bh}[\text{Mpc}^{-3}]/(d\log_{10} M_{bh}[M_{\odot}] da))$.

spin parameter $a_{bh} \sim 0.1$. This is indeed what would be expected in a chaotic-accretion scenario, where the black-hole spin oscillates around a small non-zero value (King & Pringle 2006). In our model, the value $a_{bh} \sim 0.1$ is easily explained. As mentioned in Sec. 2.2.4, accretion turns chaotic when the mass of the gaseous reservoir drops below the black hole’s mass (“dry” environment). Assuming that the MBH is almost maximally spinning as a result of

the previous phase of coherent accretion, we can calculate the spin of the MBH when the reservoir has been completely accreted by integrating Eq. (39) from an initial spin $a_{bh} = 1$, and assuming that the MBH accretes a mass of gas $M_{res} = M_{bh}^{in}$. Doing so, one gets a final spin $a_{bh} \approx 0.14$, which explains the large number of MBH with spin $a_{bh} \sim 0.1$ at low redshifts. The evolution of the spins is qualitatively similar in the heavy-seed scenario, with the difference

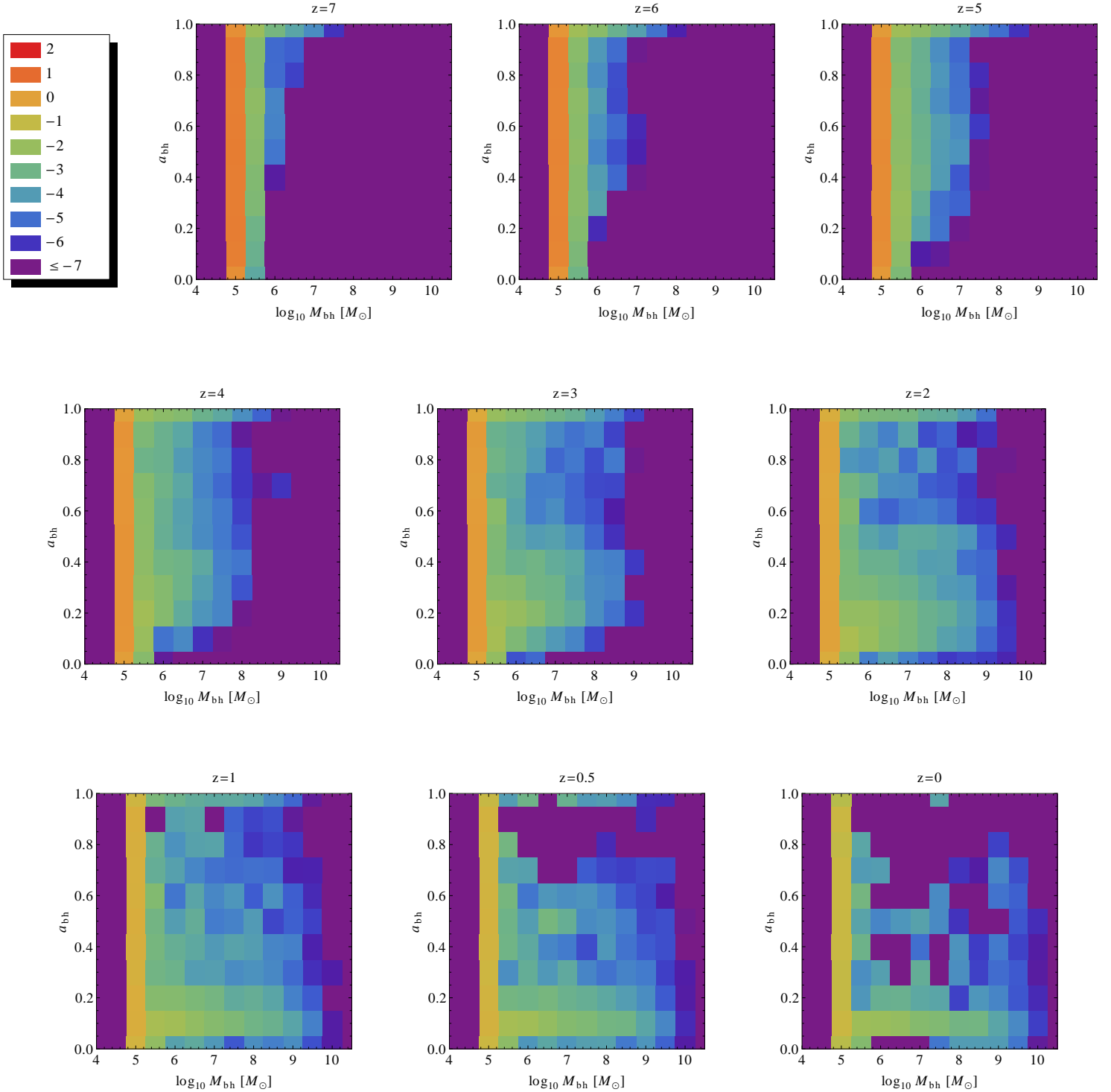


Figure 15. The same as in Fig. 14, but in the heavy-seed scenario.

that no MBHs with $M_{\text{bh}} < M_{\text{seed}} = 10^5 M_{\odot}$ are present, because of the seed-model described in Sec. 2.1.

We stress that our results, and in particular the dichotomy between almost maximal spins at high redshifts and small spins at $z \approx 0$, are qualitatively independent of our assumption that the seeds are initially assigned spin parameters drawn from a uniform distribution $-1 \leq a_{\text{bh}} \leq 1$, at least for MBH masses $M_{\text{bh}} \gtrsim 3M_{\text{seed}}$. While it is unclear whether such a spin-parameter

distribution makes sense physically, because little is known about the spins of the seeds, at high redshifts the MBHs accrete coherently, and they lose memory of their initial spin after accreting a mass comparable to their own (i.e., if a black hole of mass M_{bh} accretes coherently, its spin becomes maximal after accreting a mass $\lesssim 2M_{\text{bh}}$ (Bardeen 1970)).

In both the light and heavy-seed scenario, it would seem that the paucity of MBHs with large spins might be in contrast

with the iron $K\alpha$ measurements of the MBH spins in MCG-6-30-15 and in NGC3783, which were claimed to be respectively $a_{\text{bh}} > 0.987$ (Brenneman & Reynolds 2006) and $a_{\text{bh}} > 0.9$ (Reynolds et al. 2011; Brenneman et al. 2011) at 90% confidence level. It should be noted, however, that both these measurements are still controversial, with Patrick et al. (2011) finding $a_{\text{bh}} = 0.49^{+0.20}_{-0.12}$ for MCG-6-30-15 and $a_{\text{bh}} < -0.04$ for NGC3783, and with other iron $K\alpha$ measurements of MBH spins giving smaller values, e.g. a spin between 0.3 and 0.77, according to the measurement, for Fairall 9 (Patrick et al. 2011; Reynolds et al. 2011). Even more importantly, these spin measurements are necessarily biased by selection effects, because large spins correspond to higher emission efficiencies $\eta(a_{\text{bh}})$ and therefore higher AGN luminosities (Reynolds et al. 2011; Brenneman et al. 2011). Moreover, iron $K\alpha$ measurements are of course only possible in systems with accretion disks in the first place (i.e. in AGNs), and those systems are expected to host MBHs with high spins because coherent accretion spins them up to the maximal limit (cf. Sec. 2.2.4).

6 CONCLUSIONS

We have utilized a semianalytical galaxy-formation model to track the evolution and mergers of the dark-matter halos, the IGM, the baryonic structures (galactic disks and spheroids, in both their gaseous and stellar components), and the MBHs that are thought to reside in the center of galaxies. The evolution of the MBHs is deeply entangled with that of their host galaxies, because it is the star formation in the galactic spheroid that funnels gas to the galactic center via e.g. radiation drag. This creates a reservoir that feeds the MBH, but the character of the accretion process (coherent or chaotic) depends on the amount of gas present in this reservoir. Similarly, when two MBHs merge after their host galaxies have coalesced, their spins get aligned prior to the merger due to gravito-magnetic torques if enough gas is present in the galactic nucleus, while the orientation of the spins remains essentially isotropic in gas-poor environments. A further complication to this picture is that the MBHs also backreact on the larger-scale galactic evolution, through the so-called AGN feedback, i.e. they are thought to quench star formation in high-mass systems by injecting energy into the IGM via strong jets or accretion-disk winds. Indeed, the AGN feedback is a crucial ingredient of modern galaxy-formation models, and is needed to explain the “anti-hierarchical” evolution (or “downsizing”) of baryonic structures, i.e. the fact that the most massive galaxies are dominated by old stellar populations, while low-mass galaxies generally present young stellar populations and longer-lasting star-formation activity.

In this paper, we have made an attempt to study how this complicated interdependence between MBHs and their host galaxies affects the black-hole mass and spin evolution, considering both a scenario where MBHs form from “light” $150M_{\odot}$ seeds at $z \sim 15 - 20$ and one where they form from “heavy” $\sim 10^5 M_{\odot}$ seeds at $z \sim 10 - 15$. Besides confirming that these two scenarios may be observationally distinguishable with LISA/SGO or a similar European gravitational-wave mission (eLISA/NGO) by simply looking at the observed event rate for MBH mergers, we have studied the MBH mass and spin evolution in detail. In particular, we have determined that accretion is mostly coherent and MBH mergers happen in gas-rich environments at high redshifts, while at low redshifts, when AGN feedback, ram pressure and clumpy accretion have “sterilized” the galaxy, accretion becomes mainly chaotic and mergers happen in gas-poor environments. This results in a spin

distribution that is skewed towards large spins at high redshifts, and towards spins $a_{\text{bh}} \sim 0.1$ at low redshifts, a prediction that will be readily testable by LISA or a similar mission.

In principle, LISA/SGO or eLISA/NGO will also be capable of testing our predictions for the character of MBH mergers as function of redshift directly, because gas-rich mergers tend to present aligned spins, while gas-poor mergers tend to present randomly oriented spins, and these different orientations will produce an effect on the higher-order harmonics of the gravitational waveforms. We will examine this more in detail in future work aiming at calculating the eLISA/NGO signal-to-noise ratios and the effect of higher-order harmonics. In particular, to accurately model the gravitational waveforms for spinning black-hole binaries we will employ the effective-one-body model of Barausse & Buonanno (2010, 2011), which successfully reproduces the exact (i.e. numerical-relativity) waveforms both in the extreme-mass ratio limit (Yunes et al. 2011; Barausse et al. 2011) and in the comparable-mass case (Pan et al. 2011).

Finally we have briefly hinted at how the mass evolution of MBHs and the event rates for LISA/SGO or eLISA/NGO may be different in alternative galaxy-formation models such as the “two-phase” model of Cook, Lapi, & Granato (2009); Cook et al. (2010a,b). This model reproduces the downsizing of cosmic structures more naturally than the standard galaxy-formation model that we consider in this paper, and predicts that galactic spheroids and MBHs should grow earlier than in the “standard” model, which might result in larger event rates for LISA/SGO or eLISA/NGO (at least in the case of light MBH seeds). We will study the predictions of this alternative galaxy-formation model in detail in a future paper.

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